The mean shift update now becomes

$$\underline{m}_{y_k} = - \frac{\sum_i k' \cdot V_i}{\sum_i k'}$$

This is a tangent vector at y_k

and then we form

$$y_{k+1} = \exp_{y_k}\left(\underline{m}_{y_k}\right).$$

Notice

- in $\mathbb{R}^n$, this is just what we're used to
- for anything else, we need to be good at exp.

Some examples:

· Lie groups.

(a Lie group is a group that is a differentiable manifold, where the group's action on itself $G \times G \to G$ is differentiable; they have lots of interesting properties; all our usual matrix groups are Lie groups)

- assume a matrix representation
- define two MATRIX operators

$$e^M = \sum_{i=0}^{\infty} \frac{1}{i!} M^i$$

$$\log(M) = \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{i} \cdot (M - \mathrm{Id})^i$$

You can verify that $\log(e^M) = M$

$e^{\log M} = M$ etc.

These series may come with requirements on M for convergence!

One can now show (O'Neill, Ch 11) that

matrix tangent to group

$$\exp_X(M) = X e^{[X^{-1} \cdot M]}$$

Matrix; a point on group

if $\quad \exp_X(V) = Y \quad$ then

$$V = X \log\left[X^{-1} Y \right]$$

None of this is particularly attractive; but in special cases, we can do things

eg: $\quad SO(3) \quad$ (3D rotations)

<u>notice</u>: $\quad$ if $M(\phi)$ is $\in SO(3)$

then $\quad M^T M = \mathrm{Id}$

so $\quad M^T \nabla_\phi M + (\nabla_\phi M^T) M = 0$

now consider tangents at origin, $M = \mathrm{Id}$

$$\nabla_\phi M + \nabla_\phi M^T = 0$$

We <u>always</u> work ~~witha~~ the natural inner product on the matrix space

so if U, V are tangent vectors represented as matrices we use

$$\langle U, V \rangle = \sum_{ij} U_{ij} V_{ij}$$

$$= \mathrm{Trace}\,(U^T V)$$

So tangent space at origin consists of N st $N + N^T = 0$

write

$$[\omega]_\times = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

(here $\omega = [\omega_x, \omega_y, \omega_z]$ is axis of rotation, $\|\omega\|$ is magnitude)

then

$$e^{[\omega]_\times} = Id + \frac{\sin\|\omega\|}{\|\omega\|}[\omega]_\times \cancel{+ \frac{(1-\cos\|\omega\|)}{\|\omega\|^2}}$$

$$+ \frac{(1-\cos\|\omega\|)}{\|\omega\|^2} \cdot [\omega_\times]^2$$

(called the Rodriguez formula – expand, and match terms w/ Taylor series)

It's also straightforward to extract axis angle from a rotation. Consider $M \in SO(3)$

$$Mv = v \quad \text{if } v \text{ is } \underline{\text{axis}}$$

other two eigenvalues reveal angle; cf eigenvalues of 2D rotation.

$$\left(\lambda_1, \lambda_2 = e^{\pm i\theta} \right)$$

This seems to work rather well for clustering pose. (Subbarao + Meer paper)

Working on Grassmanians

(Earlier notes were a little sloppy; better here)

$G_{Np} = \{$ ~~Set~~ space of p-dimensional flats through the origin in $\mathbb{R}^N \}$

$\{$ which is the same as p-1 d flats in $\mathbb{R}^{n-1} \}$

Multiple representations available. We will parametrize with

$$[Y] = n \times p \text{ matrix}$$

$$\text{st} \quad Y^T Y = \mathrm{Id}.$$

Notice that multiple Y's can represent the same point on the Grassmanian

- If $M_{p \times p}$ is $p \times p$ rotation

$$M^T M = Id$$

then YM represents the same point

· Interpretation

· p orthogonal basis ~~for~~ vectors for the space we wish to rep'n

· $M_{p \times p}$, right action is a rotation of that basis w/in that space.

Tangent vectors:

· Easy argument (There's a harder argument in Edelman).

want Δ st.

$$(Y+\varepsilon\Delta)^T(Y+\varepsilon\Delta) = \mathrm{Id}$$

to first order.

so

$$Y^T\Delta = 0$$

$n \times p$

Tangent space is $p(n-p)$ dimensional, as it should be.

Metric

$$\langle \Delta, \Gamma \rangle = tr(\Delta^T \Gamma) \quad \text{as before.}$$

Then. there is a parametric form for exp

$$\exp_Y(H) = \begin{bmatrix} YV . & U \end{bmatrix} \begin{pmatrix} \cos \Sigma \\ \sin \Sigma \end{pmatrix} V^T$$

Where $U \Sigma V^T$ is SVD of H.

Proof: Edelman, p15, easy!

AND if

$$M = \exp_Y(H)$$

$$H = A \sin^{-1}(S) B^T$$

where

$$A S D^T = M - Y Y^T M$$

$$B C D^T = Y^T M$$

$$C^T C + S^T S = \mathrm{Id}$$

S, C diagonal.

(Subbarao + Meer — Havent checked this!)