Multiple views + SLAM

two important camera models

orthographic

$$\underbrace{\begin{matrix} x \\ y \\ z \end{matrix} \rightarrow \begin{matrix} x \\ y \end{matrix}}_{\text{in camera frame}}$$

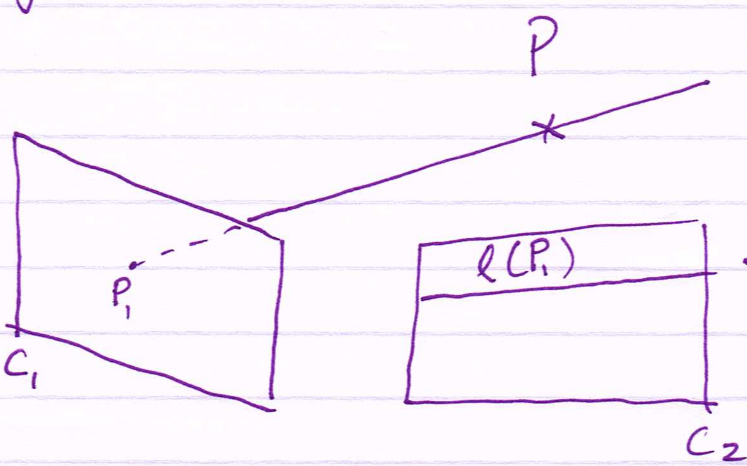
perspective

$$\begin{matrix} x \\ y \\ z \end{matrix} \rightarrow \begin{matrix} f\frac{x}{z} \\ f\frac{y}{z} \end{matrix}$$

in camera frame

Perspective cameras are often represented in Homogeneous Coordinates : (Q.V. $\xrightarrow{}$ web page)

Simplest geometry of 2 views



P could project anywhere on this line in C_2

C_1, C_2 orthographic :

1) $\ell(P_1) \parallel \ell(P_2)$ if C_2 is rot wrt C_1

2) & if no rot can't recover depth

3) Many P's in C_1 csp to the same line in C_2

4) there is a map

$$\phi : \text{pts in } C_1 \longrightarrow \text{lines in } C_2$$

5) this map:

- is rank deficient (P_1, Q_1 can give same line in C_2)
- is NOT onto
  (- there are lines in C_2 not in image(ϕ).
- follows from camera configuration

This map is useful

- constrains correspondence.
- follows from camera move

- easily estimated.

ϕ: for 2 orthographic cameras.

C_1: $\begin{matrix} x \\ y \\ z \end{matrix} \rightarrow \begin{matrix} x \\ y \end{matrix}$

C_2: $\begin{matrix} u \\ v \\ w \end{matrix} \rightarrow \begin{matrix} u \\ v \end{matrix}$ ← in camera coords

and $\begin{matrix} u \\ v \\ w \end{matrix} = R \begin{pmatrix} x \\ y \\ z \end{pmatrix} + t$

~~the in~~ line in C_1's frame:

$\begin{matrix} p_1 \\ 0 \end{matrix} + \lambda \begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$

5

in C_2's frame

$$\begin{pmatrix} r_{11} p_x + r_{12} p_y + t_x \\ r_{21} p_x + r_{22} p_y + t_y \\ r_{31} p_x + r_{32} p_y + t_z \end{pmatrix} + \lambda \begin{pmatrix} r_{13} + t_x \\ r_{23} + t_y \\ r_{33} + t_z \end{pmatrix}$$

so on C_2's image plane.

$$\begin{pmatrix} r_{11} p_x + r_{12} p_y + t_x \\ r_{21} p_x + r_{22} p_y + t_y \end{pmatrix} + \lambda \begin{pmatrix} r_{13} + t_x \\ r_{23} + t_y \end{pmatrix}$$

<u>OR</u>

$$\left[u - (r_{11} p_x + r_{12} p_y + t_x)\right](r_{23} + t_y) =$$

$$\left[v - (r_{21} p_x + r_{22} p_y + t_y)\right](r_{13} + t_x)$$

OR

$$\underbrace{\alpha}_{\text{indep } P_x, P_y} u + \overbrace{\beta}^{\text{indep } P_x, P_y} v + \underbrace{\gamma}_{\text{linear function of } P_x, P_y} = 0$$

Implies we can recover ϕ from point correspondences (without reasoning about rotation, translation).

<u>have</u>

$$\alpha p^i_{2x} + \beta p^i_{2y} + \gamma_0 + \gamma_1 p^i_{1x} + \gamma_2 p^i_{2x} = 0$$

5 unknowns <u>BUT</u> homogeneous

1 eqn per correspondence

$\therefore$ 4 points should do it.

Idea:

4 point correspondences constrain all others (if correct). But we don't know which ones are right!

Strategy

- Select 4 csps A.R. from pool of plausible csps
- Get φ
- Count inliers

- Choose the best.

- plausible:

  - (often)

    - havent moved too much
    - similar local features in pt rep'n

- inliers:

  "plausible" csps where p_2^i is "close" to line.

- How many iterations:

  - easy sum in binomial probs.

<u>Process</u>: few hyp csps + search

↓

multiple confirmed csps

↓

3D reconstruction.

3D reconstruction from <u>multiple</u> orth cameras

· <u>Simplest</u>:

<u>assume</u>:

- every camera sees every point
- csps are known
- cameras are orthographic

- Assume C.O.G of points is at origin
- notice that orth. proj. of COG is COG in camera frame.
- So translate origin in camera frame to COG
- We now need to estimate <u>only</u> camera orientations
- equiv

$$\begin{pmatrix} \cancel{x}\,u \\ \cancel{y}\,v \\ \cancel{z}\,w \end{pmatrix}_i = R_{w \to i} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

point in world C.S

No Translation!

coords in i'th camera frame.

so j'th pt in i'th camera:

$$r_{11_i} x_j + r_{12_i} y_j + r_{13_i} z_j$$

$$r_{21_i} x_j + r_{22_i} y_j + r_{23_i} z_j$$

usually, there is a scale so.

~~[illegible]~~ write $\underline{a}_i$, $\underline{b}_i$ vectors

where $\underline{a}_i^T \underline{a}_i = \underline{b}_i^T \underline{b}_i$ and

$\underline{a}_i^T \underline{b}_i = 0$

then $\underline{q}_{ij} = \begin{pmatrix} \underline{a}_i^T \underline{P}_j \\ \underline{b}_i^T \underline{P}_j \end{pmatrix}$

point j in world coords

coords of pt j in i'th camera.

Construct

$$D = \begin{bmatrix} \text{Pt 1 } x \text{ in } C_1 & \cdots & 2\ x\ C_1 & \cdots \\ 1\ \ y\ \ 1 & & 2\ y\ C_1 & \\ x\ \ 2 & & 2\ x\ C_2 & \cdots \\ y\ \ 2 & & & \\ & & & \end{bmatrix}$$

But

$$D = \begin{bmatrix} a_1^T \\ b_1^T \\ a_2^T \\ b_2^T \\ \vdots \\ \vdots \end{bmatrix} \begin{bmatrix} P_1 & P_2 & \cdots & \cdots \end{bmatrix}$$

$\Rightarrow$ SVD D ! (to 3d in this case!)

Notice important ambiguity

$$SVD(D) = U \Sigma V^T$$

→ go to 3D

$$= U_3 \Sigma_3 V_3^T$$

write $A = U_3 \Sigma_3^{1/2}$ $B = \Sigma_3^{1/2} V_3^T$

Now $AB \approx D$

but $A W W^{-1} B \approx D$ ← same error.

But $a_i^T b_i = 0$, $a_i^T a_i = b^T b_i$

⇒ search for W st AW

has this property.

This is the root of a very rich enterprise; <u>issues</u>

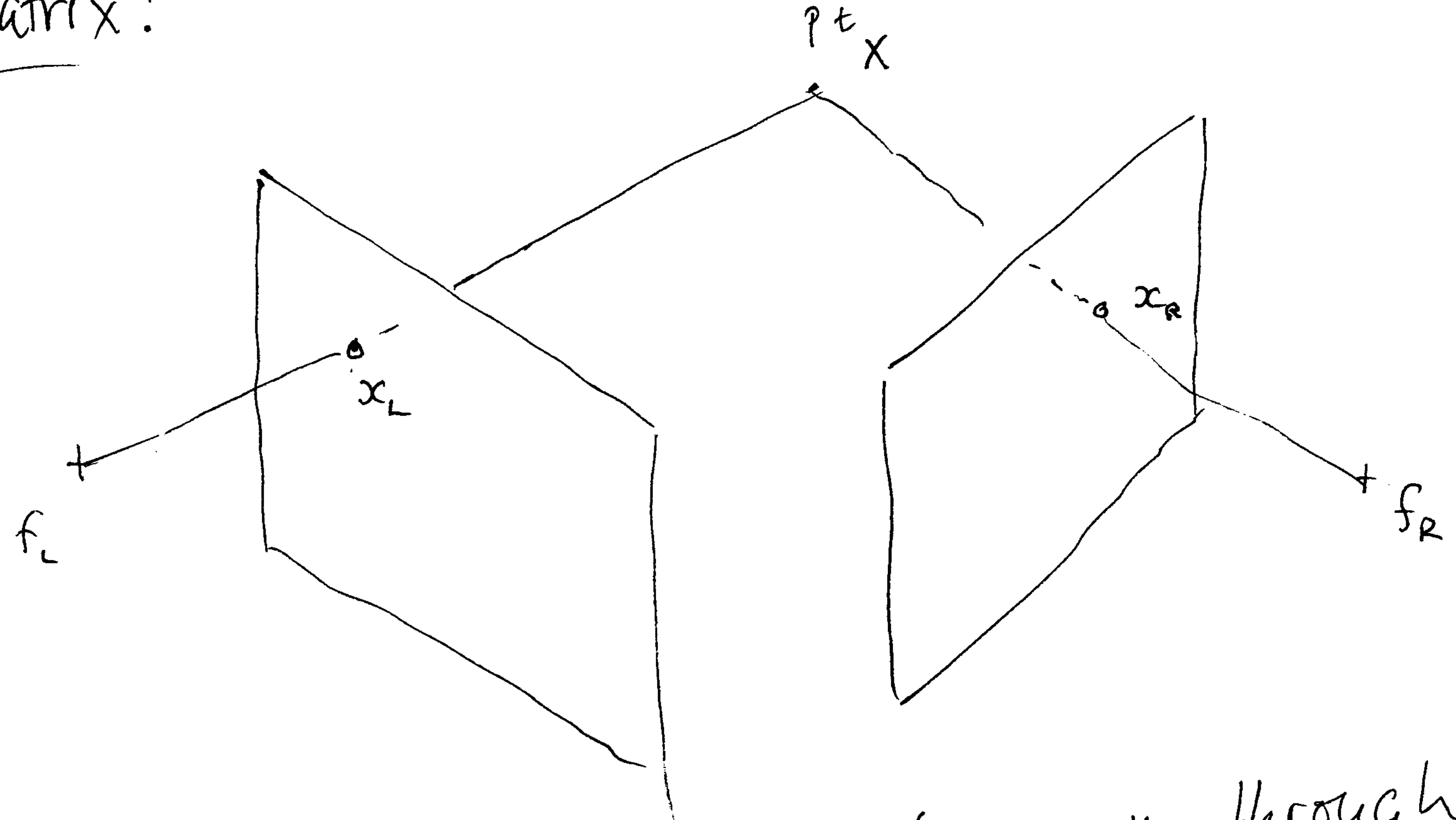
- What if matches don't appear in all frames?
- What if cameras are not orthographic
- What if views are far apart?
- What if you don't have points?
- What if cameras aren't calibrated?

- Good answers available to all - we will explore some.

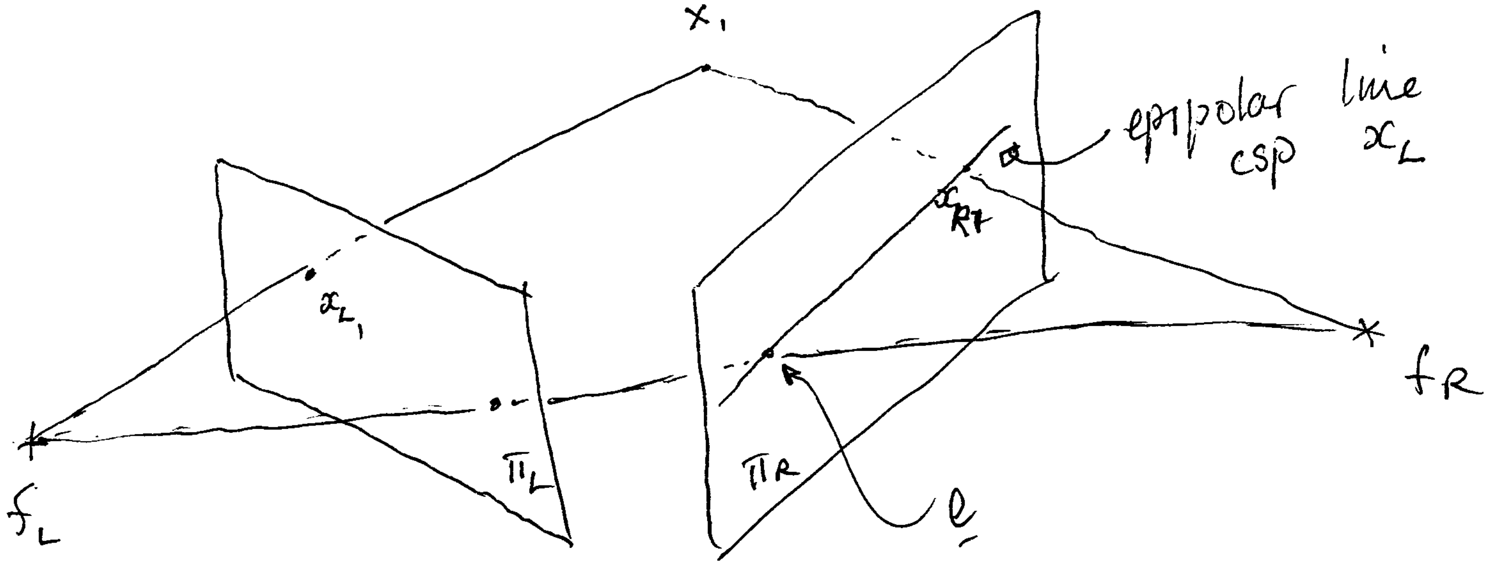
Perspective cameras:

- we cannot recover all camera translation for orth. cameras (think about sliding along rays)
- but we can for persp. cameras

Epipoles, epipolar structure and the fundamental matrix:



Notice: as X moves along ray through x_L, x_L fixed, x_R along line (as in orthographic case)



(x_{L_1}, X_1, x_{R_1}) form a plane.

$\equiv$

(f_{L_1}, X_1, f_{R_1})'s plane.

$x_{L_1} \rightarrow$ (line in R, given by plane $\cap$ π_R)

$\hookrightarrow$ epipolar line

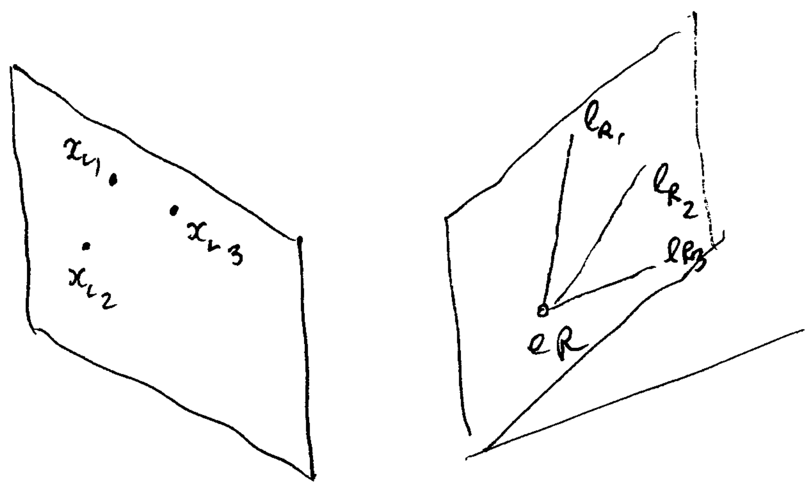
now

$x_{L_1} \longrightarrow l_{R_1} \leftarrow$ line

$x_{L_2} \longrightarrow l_{R_2} \leftarrow$ line

$l_{R_1} \cap l_{R_2}$ at $\underline{e}_R$

for <u>any</u> $\cancel{l_R}$ x_{L_1}, x_{L_2}



$\underline{e}_R \leftarrow$ the epipole (in right image) where line(f_L, f_R) intersects right image plane.

<u>Notice</u> l_{R}, can be written as

$$ax + by + c = 0$$ ⟵ <u>if's a line</u>

$$\begin{bmatrix} \text{fn of } x_e, \\ f_L, f_R \end{bmatrix} \longrightarrow$$ <u>linear</u> in x_L because of how plane eq. is formed.

So there is a <u>matrix</u> F

st

$$x_L^T F x_R = 0$$

where $x_L = \begin{pmatrix} x_{L1} \\ x_{L2} \\ 1 \end{pmatrix}$ etc

OR Homogeneous coords.

This matrix

Why? It maps ~~3D~~ 2D family of pts $\longrightarrow$ 1D family of lines