$$\underline{n} \xleftarrow{\text{rescale, translate}} \underline{u} \xleftarrow{\text{Project}} \underline{U} \xleftarrow{\text{Rotate, Translate}} \bullet\ X$$

X in world coords

$\underline{U}$ in camera centered coords, fp at origin

$$\begin{matrix} U_x \\ U_y \\ U_z \end{matrix} \longrightarrow \begin{matrix} \frac{U_x}{U_z} \\ \frac{U_y}{U_z} \end{matrix}$$

$\underline{n}$: numbers that come out of camera.

pixels/μ

$$\underline{n} = \underbrace{\begin{pmatrix} \frac{s}{f} & 0 \\ 0 & \frac{as}{f} \end{pmatrix}}_{K} \underline{u} + \begin{pmatrix} c_x \\ c_y \end{pmatrix}$$

aspect ratio

focal length

principal point, camera center

K, c are <u>intrinsics</u>

- can be recovered by calibrati[on]
- other methods are possible

Recovering

$$\mathrm{SVD}(\mathcal{E}) = U^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V.$$

Now :
$$W = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad Z = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and
$$ZW = \mathrm{diag}[1, 1, 0]$$
$$ZW^T = -\mathrm{diag}[1, 1, 0]$$

So: and W is a rotation ~~(note~~

$U W V^T$ is a rot.

so

$U W^T V^T$ is a rot.

then
$$E = (-UZU^T)(UW^TV^T)$$
$$= (UZU^T)(UWV^T)$$

check these are antisymmetric | rots, distinct !

Recovering

how we need a translation vector.

recall $$T_x = \begin{pmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{pmatrix}$$

ie. $T_x \underline{t} = 0$

so $\cancel{t = 0}$ $$U Z U^T t = 0$$

$$Z \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$\therefore \quad U^T t = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$t = \begin{pmatrix} u_{31} \\ u_{32} \\ u_{33} \end{pmatrix}$$

But if $T_x t = 0$

$$T_x (\lambda t) = 0 \quad !$$

Choice of λ ...

- λE is also an essential matrix ($\lambda \neq 0$) $\therefore$ can't get scale
- Sign of λ is important; are points in front / behind camera.
- So: four solns

R	t
UWV^T	u_3
UWV^T	$-u_3$
UW^TV^T	u_3
UW^TV^T	$-u_3$

- all points lie on plane
- Considerable simplification follows.

plane $\quad z = ax + by + c$

(in Camera 1's coords.

C_1 reports $\quad \dfrac{x}{ax+by+c}, \dfrac{y}{ax+by+c}$

in HC's

$$\begin{pmatrix} 1 & & \\ & 1 & \\ a & b & c \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

C_2 reports

$$C_1 \circ (Rot, tx)$$

$$\begin{pmatrix} r_{11} + r_{13}a & r_{12} + r_{13}b & t_x \\ r_{21} + r_{23}a & r_{22} + r_{23}b & t_y \\ r_{31} + r_{33}a & r_{32} + r_{33}b & t_z \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$\therefore$ T_x $\quad C_1 \longrightarrow C_2$.

$$\underbrace{\begin{pmatrix} & & t \\ r_1 & & \\ & & \end{pmatrix}\begin{pmatrix}1 & & \\ & 1 & \\ a & b & c\end{pmatrix}^{-1}} \longrightarrow \text{homograph}$$

<u>Strategy</u>:

- compute homography
- recover parameters (papers)