Cameras

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Cameras

- First photograph due to Niepce
- First on record shown in the book - 1822
- Basic abstraction is the pinhole camera
  - lenses required to ensure image is not too dark
  - various other abstractions can be applied
Pinhole cameras

- Abstract camera model - box with a small hole in it
- Pinhole cameras work in practice
Distant objects are smaller
Parallel lines meet

Common to draw film plane *in front* of the focal point.
Moving the film plane merely scales the image.
Vanishing points

• Each set of parallel lines (=direction) meets at a different point
  • The vanishing point for this direction

• Sets of parallel lines on the same plane lead to collinear vanishing points.
  • The line is called the horizon for that plane

• Good ways to spot faked images
  • scale and perspective don’t work
  • vanishing points behave badly
  • supermarket tabloids are a great source.
The equation of projection - I
The equation of projection - II

- Focal point
- Image plane
- Object Height = X
- Image Height = x
- Z axis
- Z
- f
The equation of projection - III

• Cartesian coordinates:
  • We have, by similar triangles, that \((X, Y, Z) \rightarrow (fX/z, fY/z, f)\)
  
• Ignore the third coordinate, and get
  \[(X, Y, Z) \rightarrow (fX/Z, fY/Z)\]

• notice we could have sign changes, etc. depending on
  • whether there is a right handed/left handed coordinate system
  • whether image plane is in front of/behind focal point
Homogenous coordinates

- Add an extra coordinate and use an equivalence relation
- for 2D
  - equivalence relation
    \[ k*(X,Y,Z) \text{ is the same as } (X,Y,Z) \]
- for 3D
  - equivalence relation
    \[ k*(X,Y,Z,T) \text{ is the same as } (X,Y,Z,T) \]
- Basic notion
  - Possible to represent points “at infinity”
  - Where parallel lines intersect
  - Where parallel planes intersect
- Can write the action of a perspective camera as a matrix
The camera matrix

- Turn previous expression into HC’s
  - HC’s for 3D point are (X,Y,Z,T)
  - HC’s for point in image are (U,V,W)

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} =
\begin{pmatrix}
f & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
X \\
Y \\
Z \\
T
\end{pmatrix}
\]
The camera matrix - II

- Turn previous expression into HC’s
  - HC’s for 3D point are \((X,Y,Z,T)\)
  - HC’s for point in image are \((U,V,W)\)

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} = C \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{pmatrix} \begin{pmatrix}
X \\
Y \\
Z \\
T
\end{pmatrix}
\]

- Transforms points from object coordinates into world coordinates most likely a rotation and translation
- Transforms camera coordinates (f is hidden in here)
Usual forms

Camera intrinsic parameters

\[ C = \begin{pmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{pmatrix} \]

Pixel aspect ratio

A scale, incorporating focal length and pixel size

Location of the camera center (where the z-axis pierces the image plane)

3D rotation matrix 3D translation vector

\[ W = \begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix} \]
Orthographic projection
The projection matrix for orthographic projection

- Almost never encounter orthographic projection

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} = C \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \mathcal{W} \begin{pmatrix}
X \\
Y \\
Z \\
T
\end{pmatrix}
\]
Weak perspective or Affine Cameras

• Assume
  • The range of depths over points is small compared to distance to points

\[ Z_i = Z_0 + \delta Z_i \]

\[ \frac{1}{Z_i} = \frac{1}{Z_0} \left( \frac{1}{1 + \frac{\delta Z_i}{Z_0}} \right) \approx \frac{1}{Z_0} (1 - \frac{\delta Z_i}{Z_0}) \approx \frac{1}{Z_0} \]

• So you could scale all points with one scale
  • Scaled orthography
Affine Cameras - I

• And this becomes (for the relevant group of points)

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} = C \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
X \\
Y \\
Z \\
T
\end{pmatrix}
\]

Hide the scale in here

• We will see further simplifications soon
Pinhole too big - many directions are averaged, blurring the image

Pinhole too small - diffraction effects blur the image

Generally, pinhole cameras are *dark*, because a very small set of rays from a particular point hits the screen.
The reason for lenses
Lenses come with problems

- Spherical aberration
  - Lens is not a perfect thin lens, and point is defocused
Lens systems
Vignetting
Other (possibly annoying) phenomena

- Chromatic aberration
  - Light at different wavelengths follows different paths; hence, some wavelengths are defocussed
  - Machines: coat the lens
  - Humans: live with it

- Scattering at the lens surface
  - Some light entering the lens system is reflected off each surface it encounters (Fresnel’s law gives details)
  - Machines: coat the lens, interior
  - Humans: live with it (various scattering phenomena are visible in the human eye)

- Geometric phenomena (Barrel distortion, etc.)
Camera calibration - I

- **Issues:**
  - what are intrinsic parameters of the camera?
  - what is the camera matrix? (intrinsic+extrinsic)

- **General strategy:**
  - view calibration object; identify image points in image
  - obtain camera matrix by minimizing error
  - obtain intrinsic parameters from camera matrix
Camera calibration - II

- **Error minimization:**
  - Linear least squares
    - easy problem numerically
    - solution can be rather bad
  - Minimize image distance
    - more difficult numerical problem
    - solution usually rather good
    - Numerical scaling is an issue
- **Strategy:**
  - start with linear least squares, then minimize image distance
Cameras - crucial points

- Pinhole camera is a simple, effective model
  - With important effects
    - distant objects are smaller
    - parallel lines meet in image
- Alternative model: orthographic projection
  - distant objects are not smaller
  - parallel lines do not meet
- Each has straightforward mathematical form
- Most cameras have lenses
  - otherwise they’d be too dark