Motion Planning I

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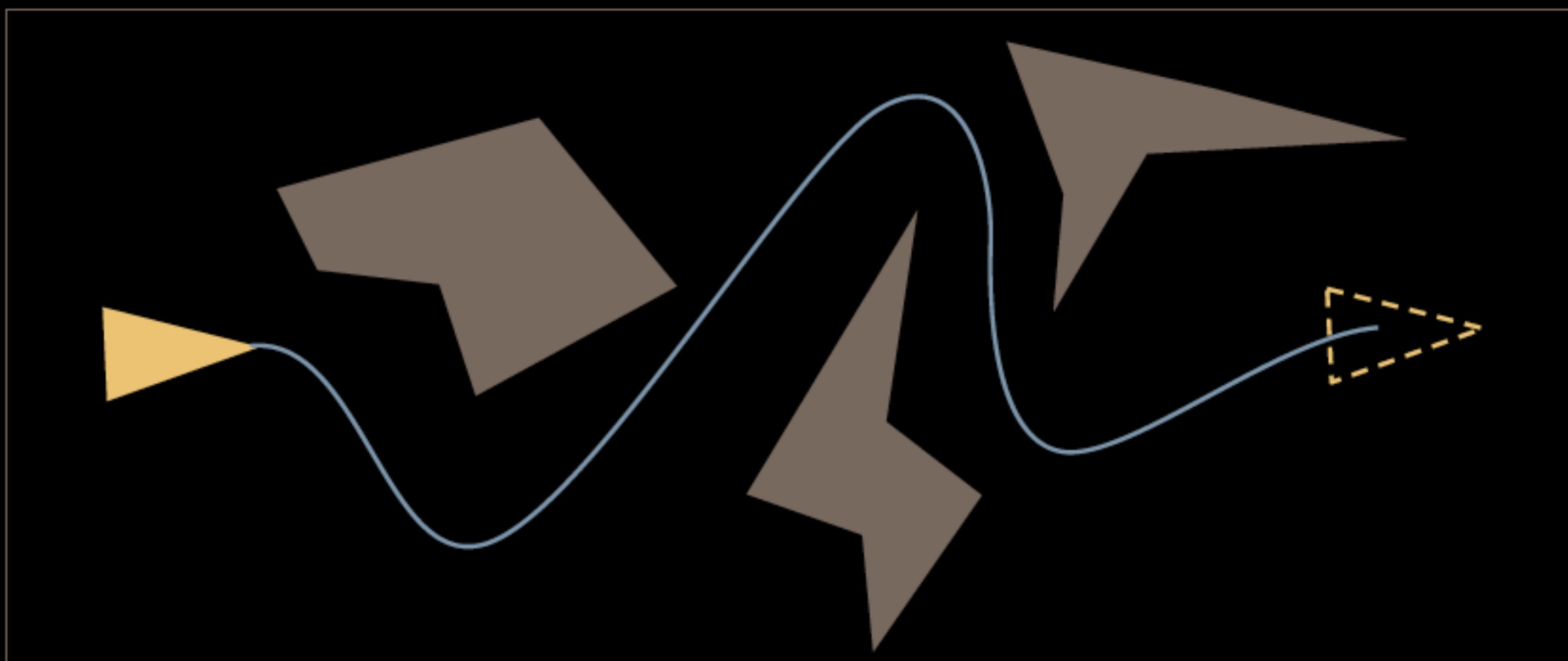
(with a lot of H. Choset, and some J. Li)

What is motion planning?

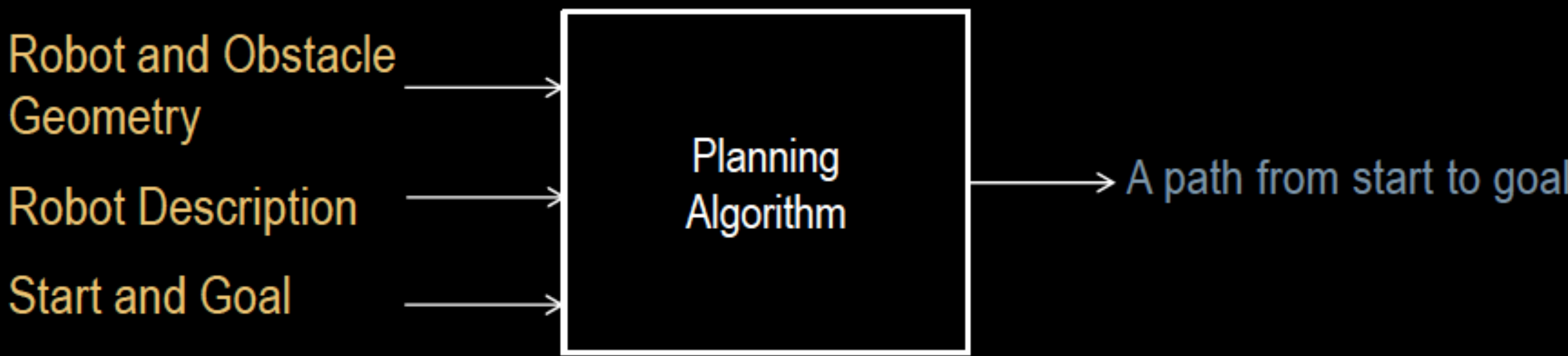
- The automatic generation of motion
  - Path + velocity and acceleration along the path



Li slides

Basic Problem Statement

- Motion planning in robotics
  - Automatically compute a path for an object/robot that does not collide with obstacles.



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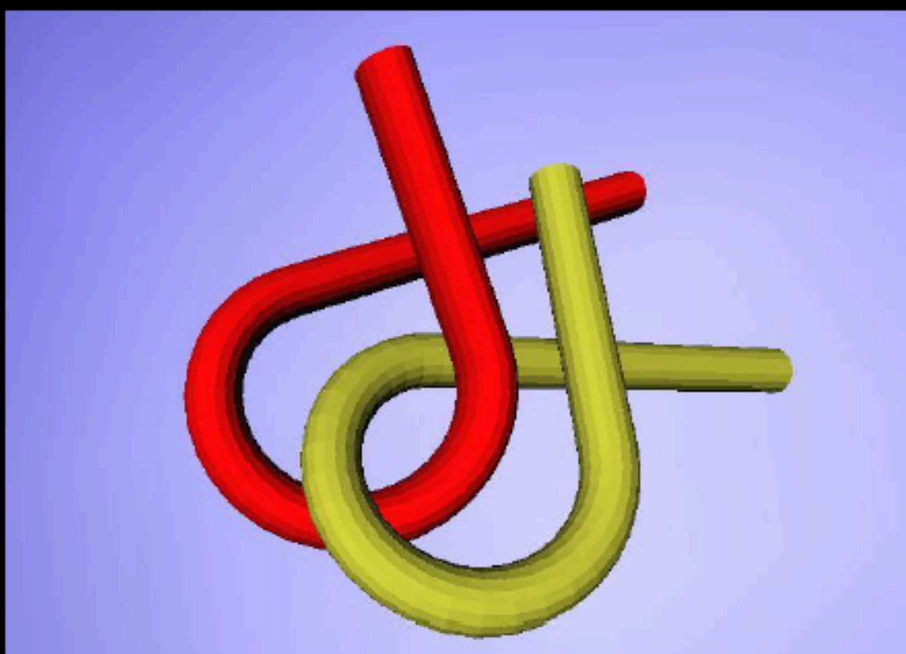
Why is this not just optimization?

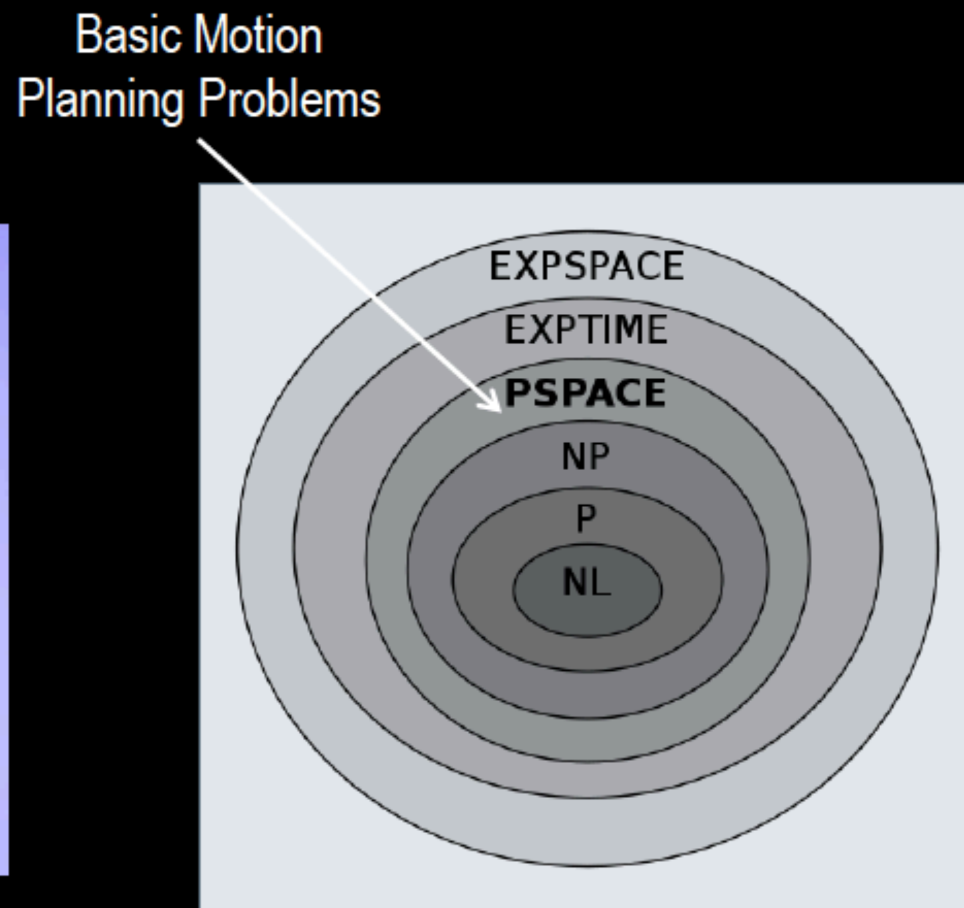
- Find minimum cost set of controls that
  - take me from A to B
  - do not involve
    - collision
    - unnecessary extreme control inputs
    - unnecessary extreme behaviors

These will have to deal with collisions, etc. →

$$
\begin{aligned}
&\text{minimize } f(\mathbf{x}) && \text{(1a)}\\
&\text{subject to} && \text{(1b)}\\
&\quad g_i(\mathbf{x}) \le 0, \quad i = 1, 2, \ldots, n_{ineq} && \text{(1c)}\\
&\quad h_i(\mathbf{x}) = 0, \quad i = 1, 2, \ldots, n_{eq} && \text{(1d)}
\end{aligned}
$$

Is motion planning hard?





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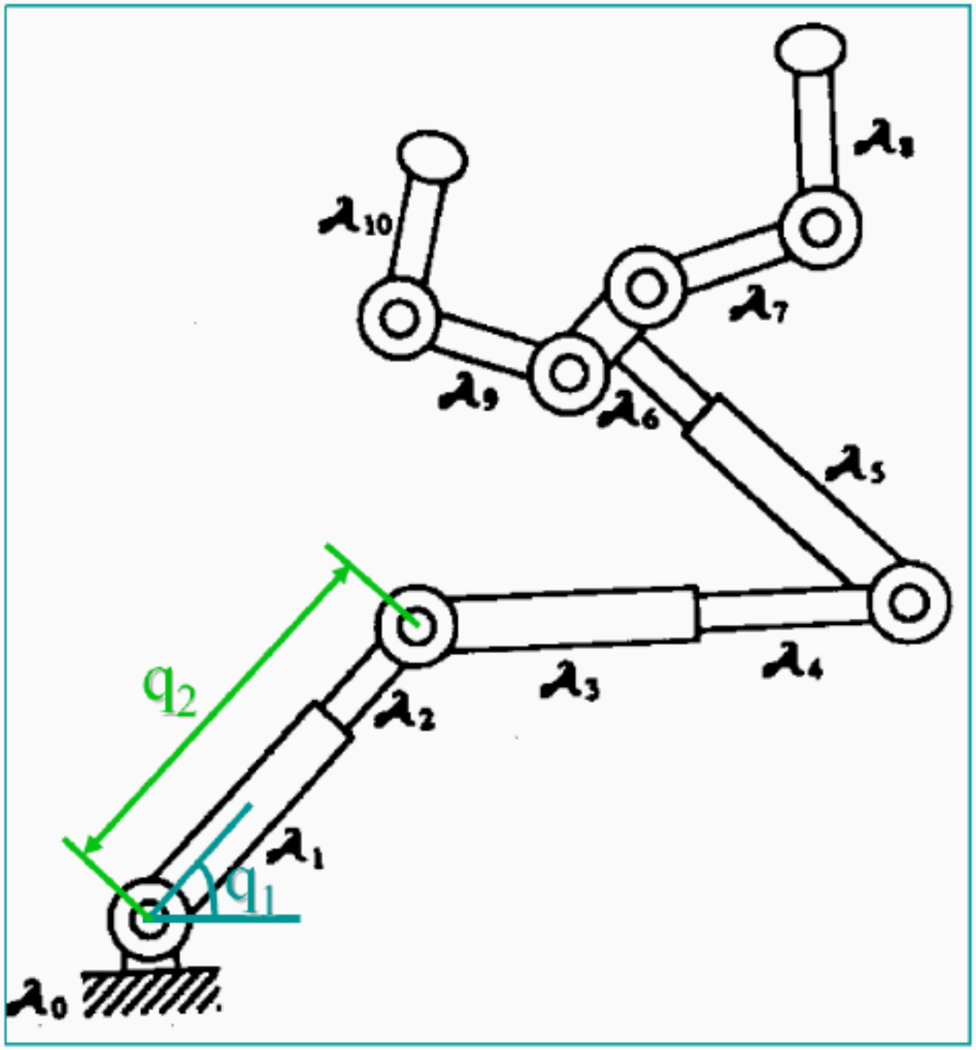
Degrees of Freedom



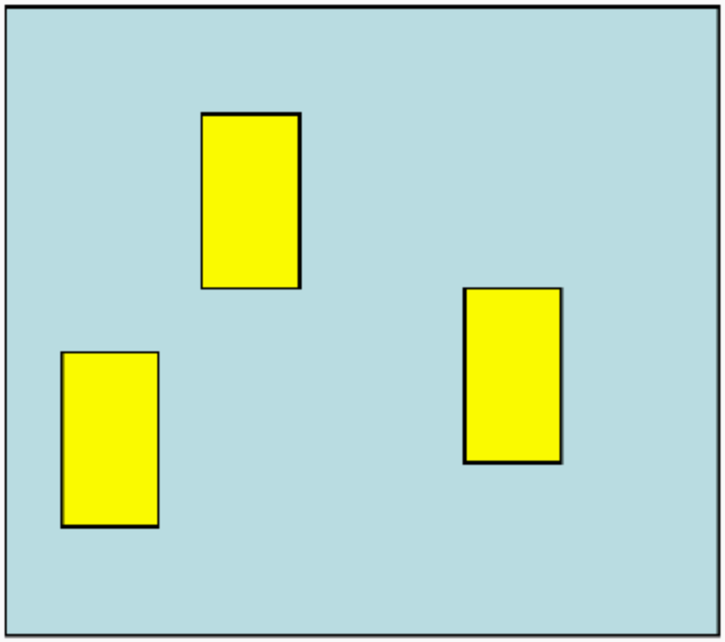
- The geometric configuration of a robot is defined by p degrees of freedom (DOF)
- Assuming p DOFs, the geometric configuration A of a robot is defined by p variables:

$A(\boldsymbol{q})$ with $\boldsymbol{q} = (q_1,\ldots,q_p)$

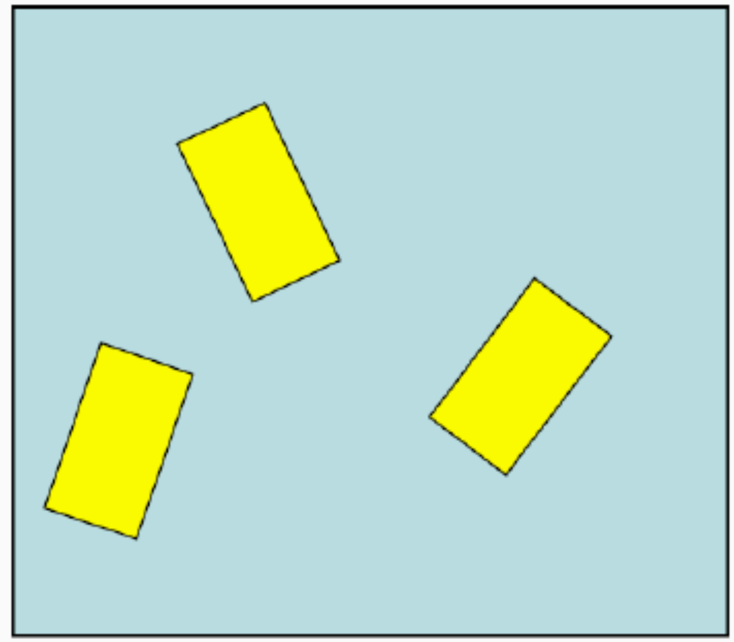
- Examples:
  - Prismatic (translational) DOF: q_i is the amount of translation in some direction
  - Rotational DOF: q_i is the amount of rotation about some axis

Our car has 3

Examples

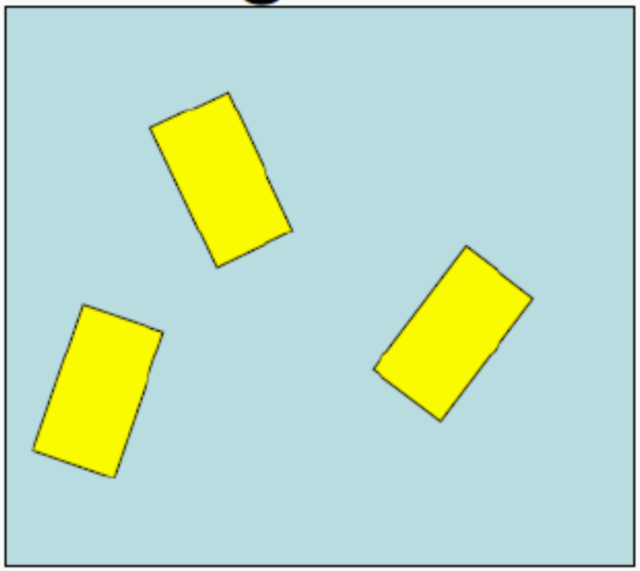


Allowed to move only in *x* and *y*: 2DOF



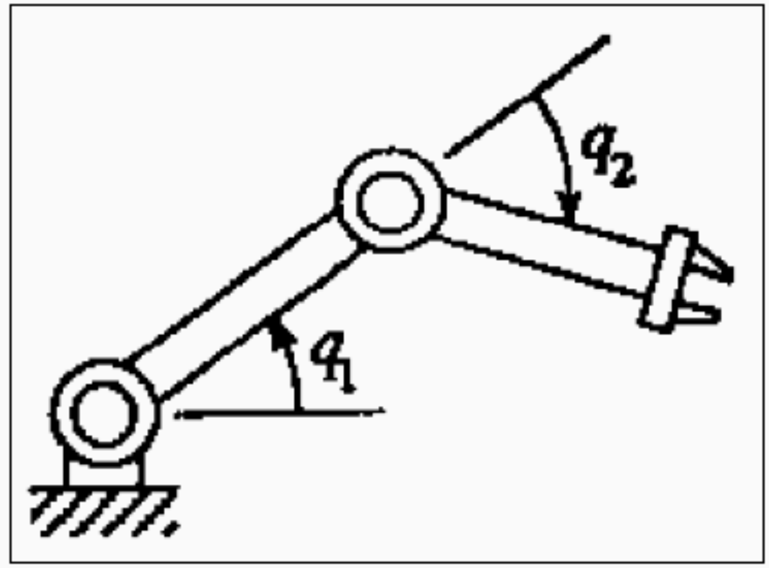
Allowed to move in *x* and y and to rotate: 3DOF (x,y,θ)

Configuration Space (C-Space)



$\boldsymbol{q} = (x, y, \theta)$

$\mathcal{C} = \Re^2 \times$ set of 2-D rotations



$\boldsymbol{q} = (q_1, q_2)$

$\mathcal{C} =$ 2-D rotations x 2-D rotations

- Configuration space $\mathcal{C}$ = set of values of $\boldsymbol{q}$ corresponding to legal configurations of the robot
- Defines the set of possible parameters (the search space) and the set of allowed paths

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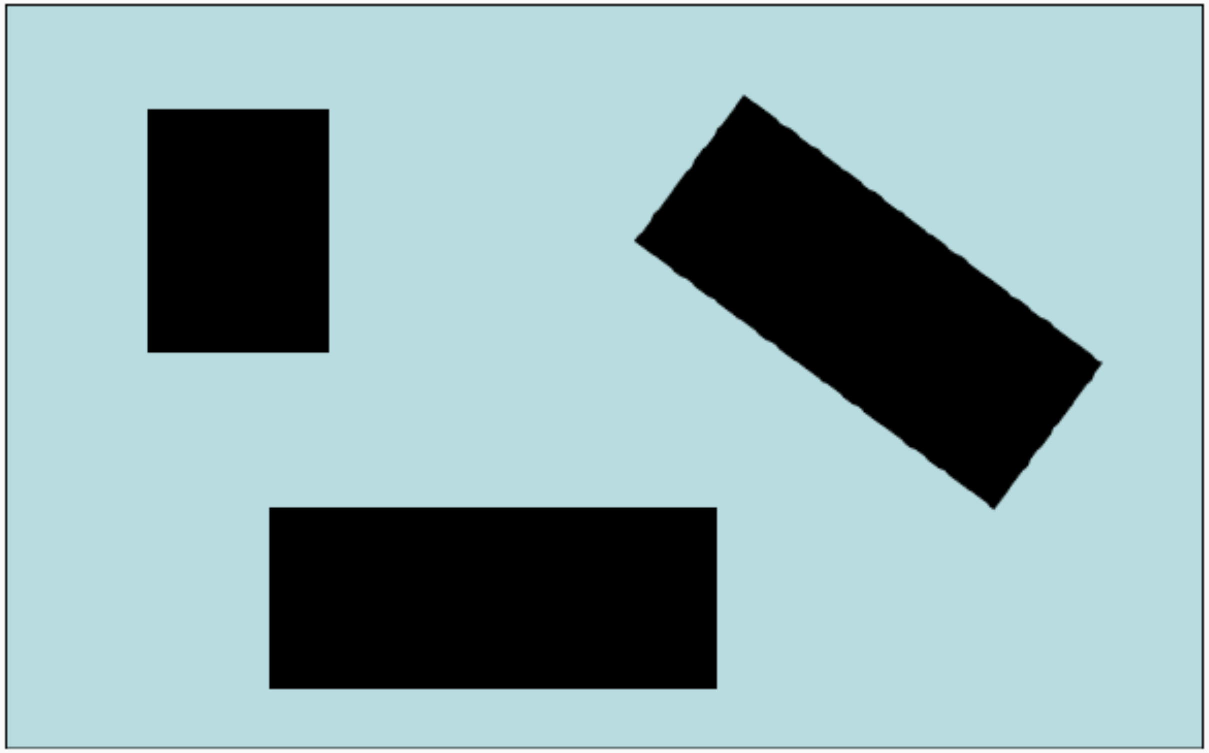
Free Space: Point Robot



- $\mathcal{C}_{free}$ = {Set of parameters $\boldsymbol{q}$ for which $A(\boldsymbol{q})$ does not intersect obstacles}
- For a point robot in the 2-D plane: R^2 minus the obstacle regions

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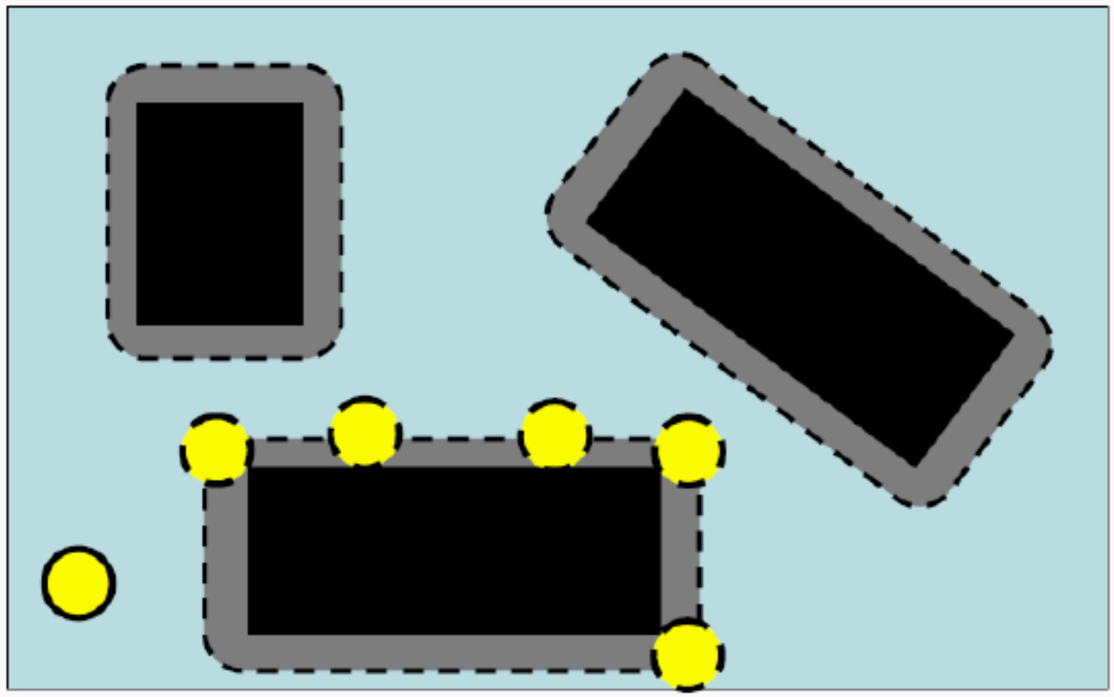
Free Space: Symmetric Robot



- We still have $\mathcal{C} = R^2$ because orientation does not matter
- Reduce the problem to a point robot by expanding the obstacles by the radius of the robot

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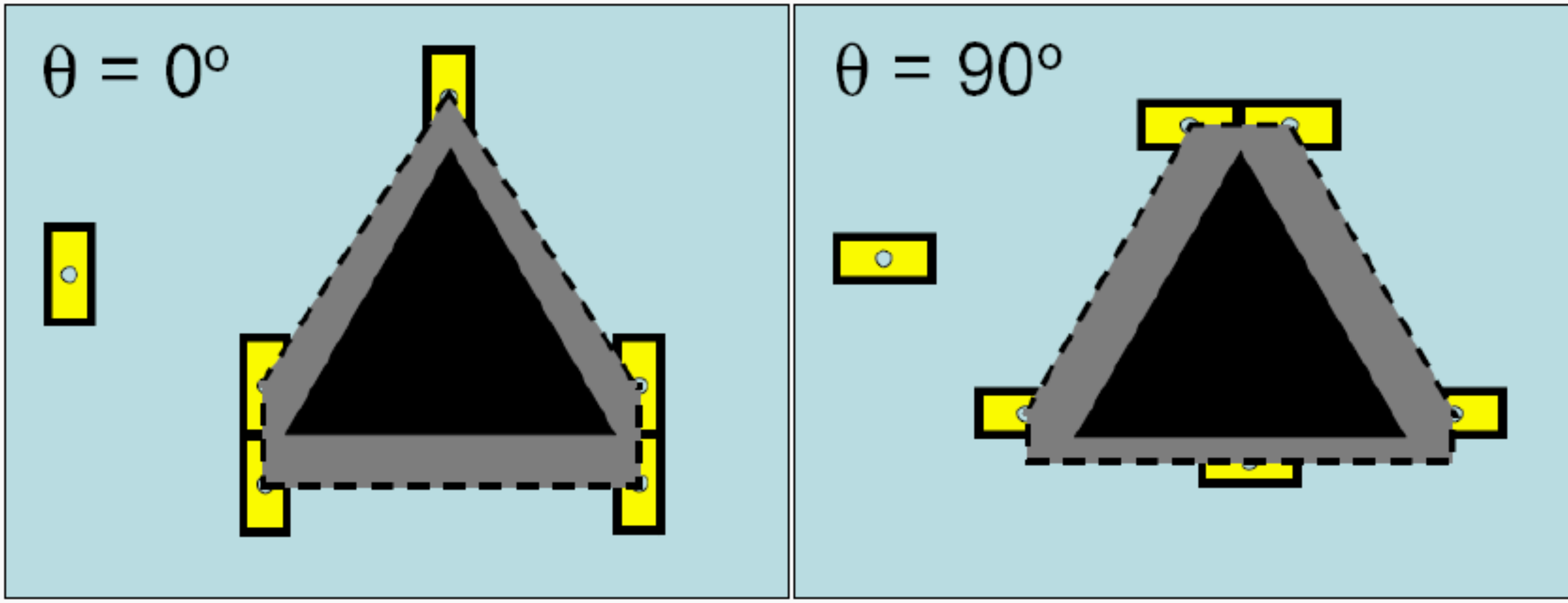
Free Space: Non-Symmetric Robot



- The configuration space is now three-dimensional (x, y, θ)
- We need to apply a different obstacle expansion for each value of θ
- We still reduce the problem to a point robot by expanding the obstacles

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Any Formal Guarantees? Generic Piano Movers Problem

- Formal Result (but not terribly useful for practical algorithms):
  - *p*: Dimension of $\mathcal{C}$
  - *m*: Number of polynomials describing $\mathcal{C}_{free}$
  - *d*: Max degree of the polynomials
- A path (if it exists) can be found in time *exponential* in *p* and *polynomial* in *m* and *d*

[From J. Canny. "The Complexity of Robot Motion Planning Plans". MIT Ph.D. Dissertation. 1987]

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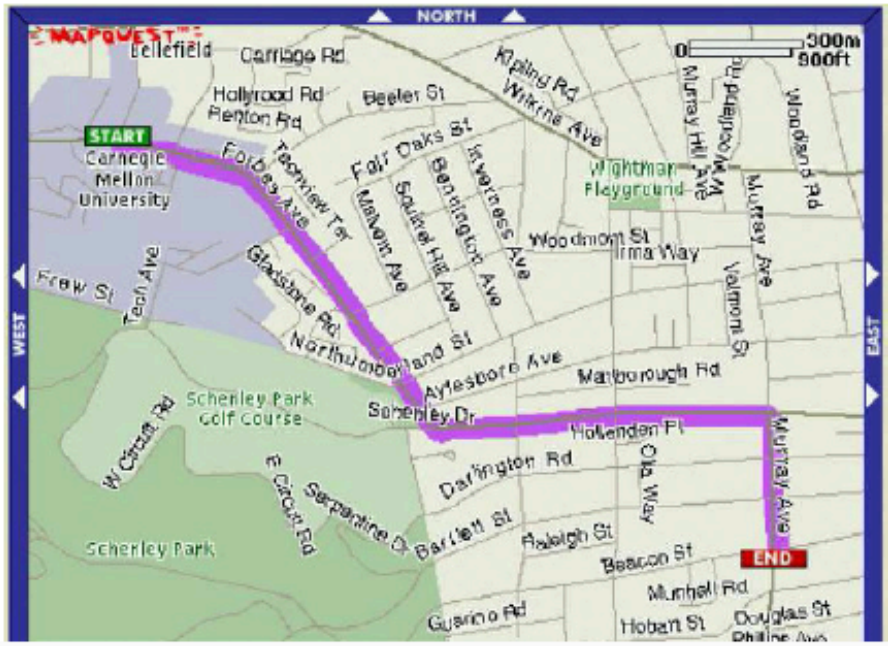
Observation

- Generally, searching a graph is pretty straightforward
  - Dijkstra, A*, etc - know how to do this
- Strategy
  - get a graph we can search

Roadmaps



- General idea:
  - Avoid searching the entire space
  - Pre-compute a (hopefully small) graph (the roadmap) such that staying on the “roads” is guaranteed to avoid the obstacles
  - Find a path between $\boldsymbol{q}_{start}$ and $\boldsymbol{q}_{goal}$ by using the roadmap

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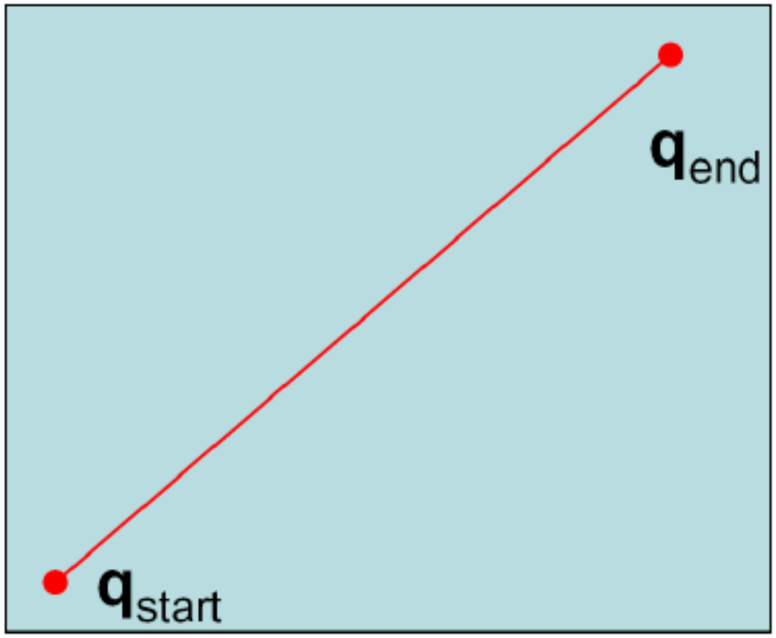
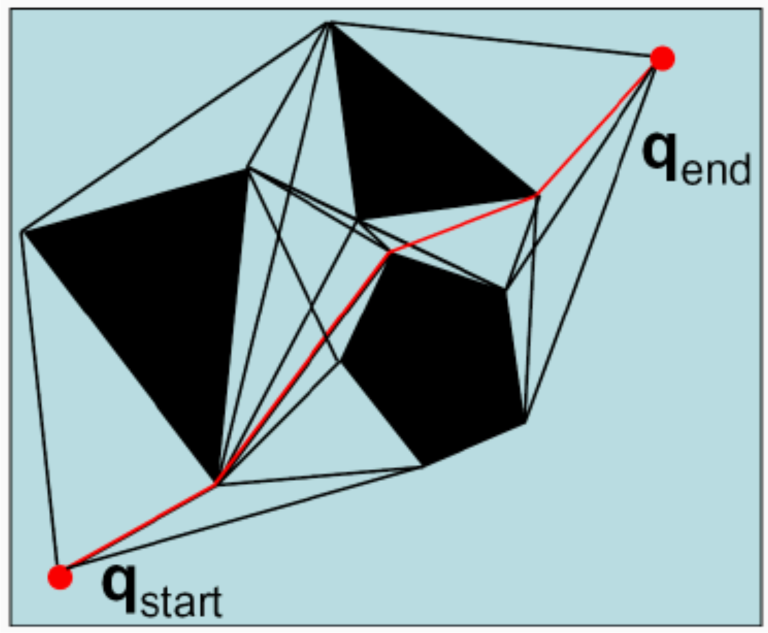
Visibility Graphs



In the absence of obstacles, the best path is the straight line between $\mathbf{q}_{start}$ and $\mathbf{q}_{goal}$

Visibility Graphs



- Visibility graph G = set of unblocked lines between vertices of the obstacles + $\mathbf{q}_{start}$ and $\mathbf{q}_{goal}$
- A node P is linked to a node P' if P' is visible from P
- Solution = Shortest path in the visibility graph

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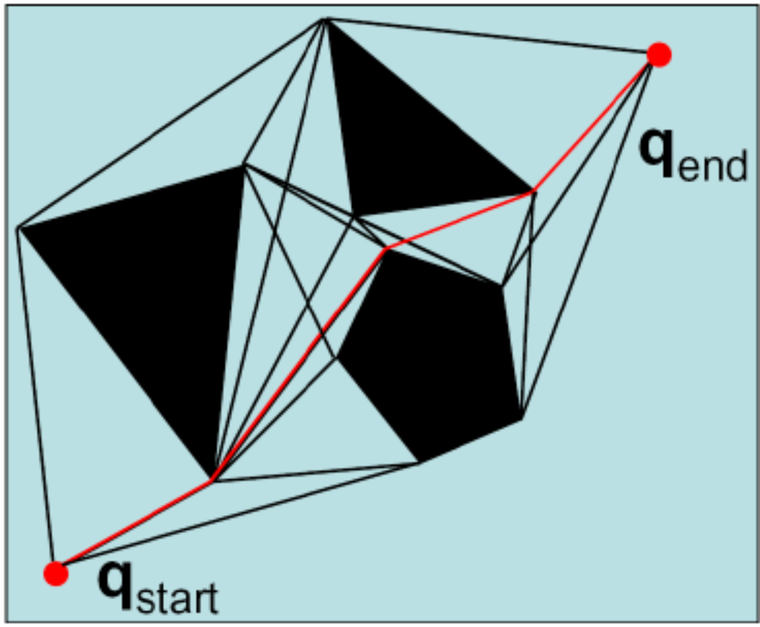
Issues

- Constructing
  - Relatively straightforward with a sweep algorithm
    - Variant (visibility complex) root cause of early computer games
      - Wolfenstein 3D, Doom II, etc
- What if configuration space is not 2D
  - You can still construct, MUCH harder
- MANY locally optimal paths
  - topology of free space clearly involved



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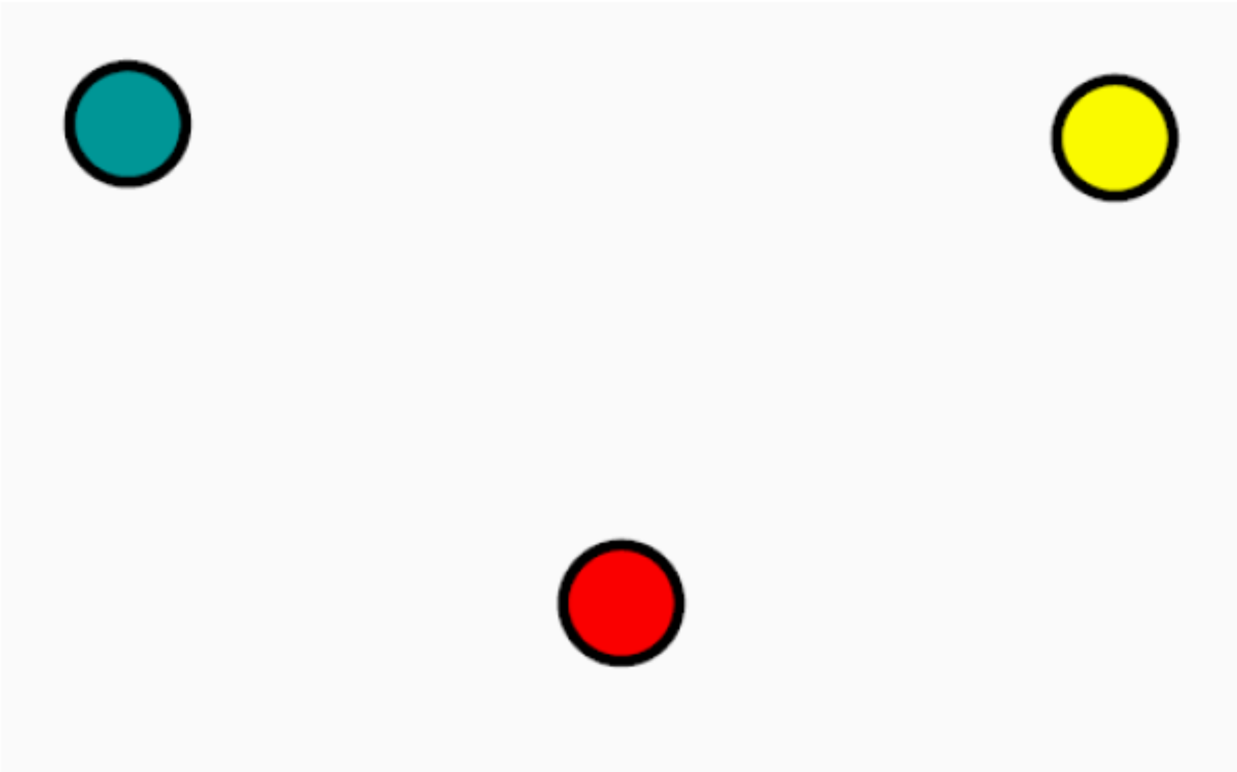
Visibility Graphs: Weaknesses

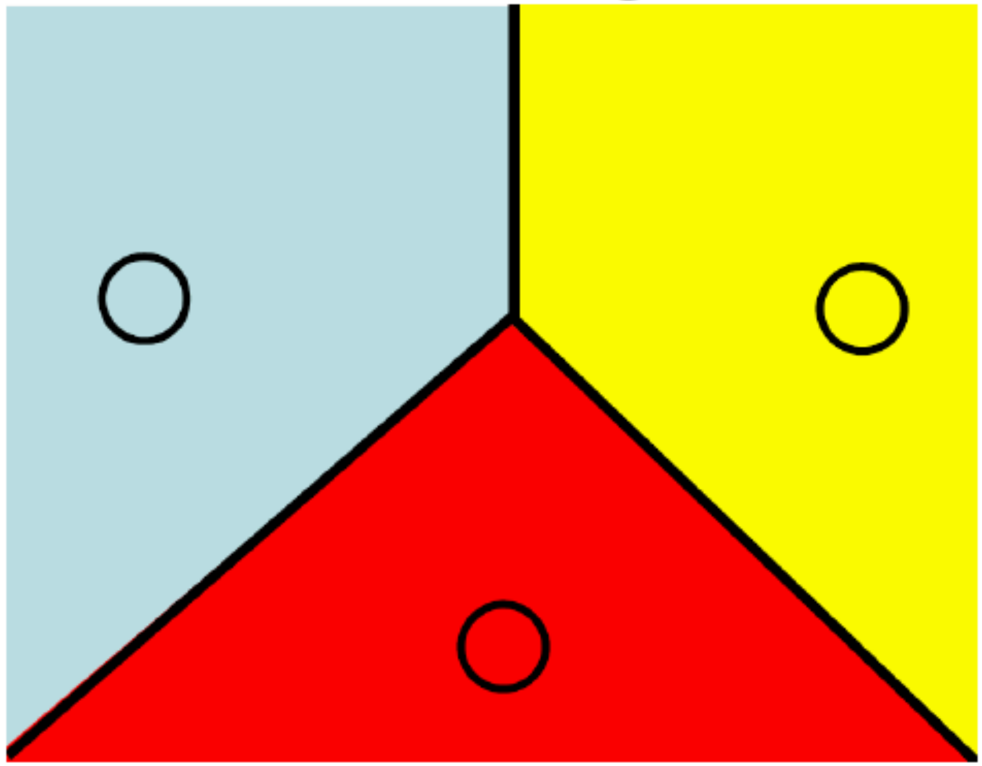
- Shortest path but:
  - Tries to stay as close as possible to obstacles
  - Any execution error will lead to a collision
  - Complicated in >> 2 dimensions
- We may not care about strict optimality so long as we find a safe path. Staying away from obstacles is more important than finding the shortest path
- Need to define other types of “roadmaps”

Voronoi Diagrams



- Given a set of data points in the plane:
  - Color the entire plane such that the color of any point in the plane is the same as the color of its nearest neighbor

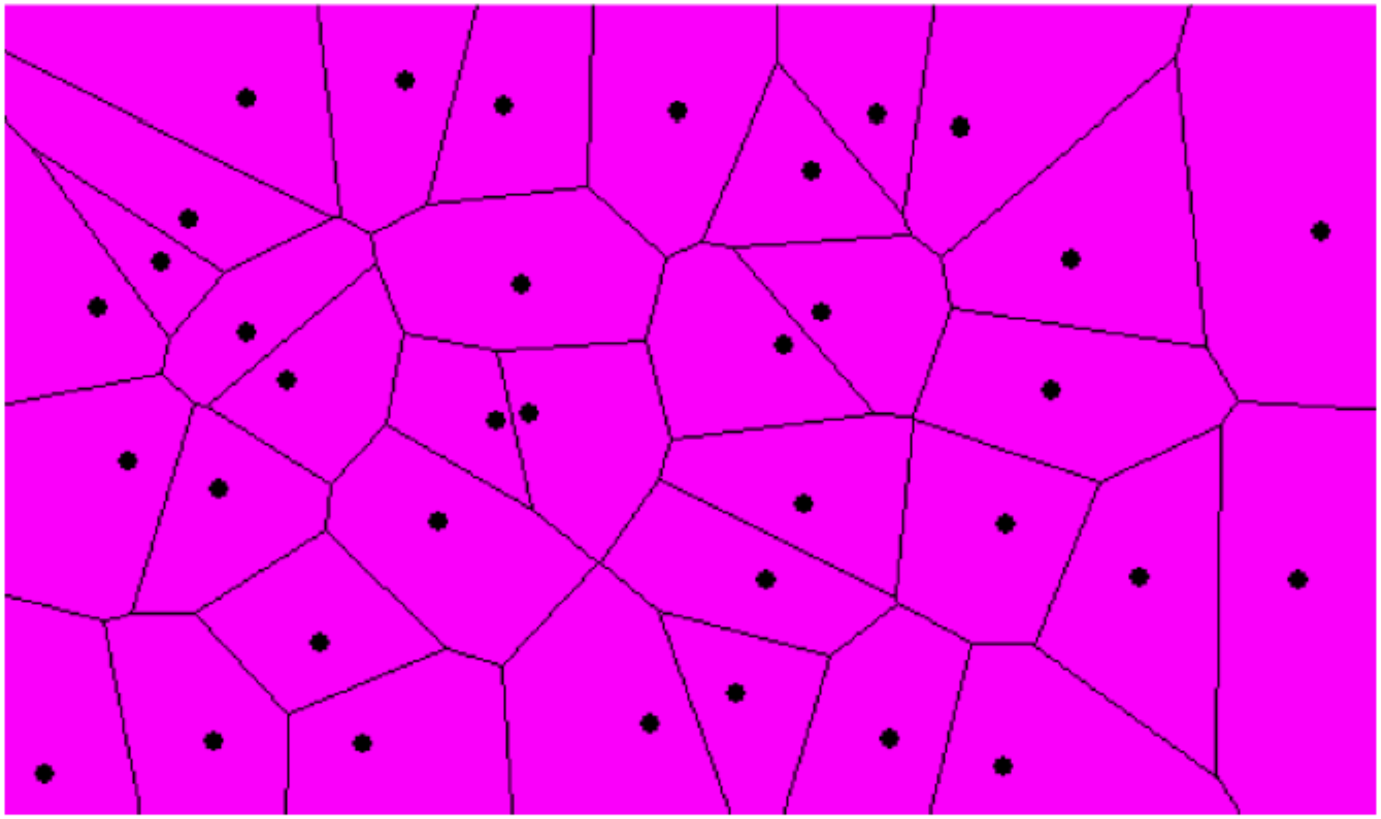
Voronoi Diagrams



• Voronoi diagram = The set of line segments separating the regions corresponding to different colors

- Line segment = points equidistant from 2 data points
- Vertices = points equidistant from > 2 data points

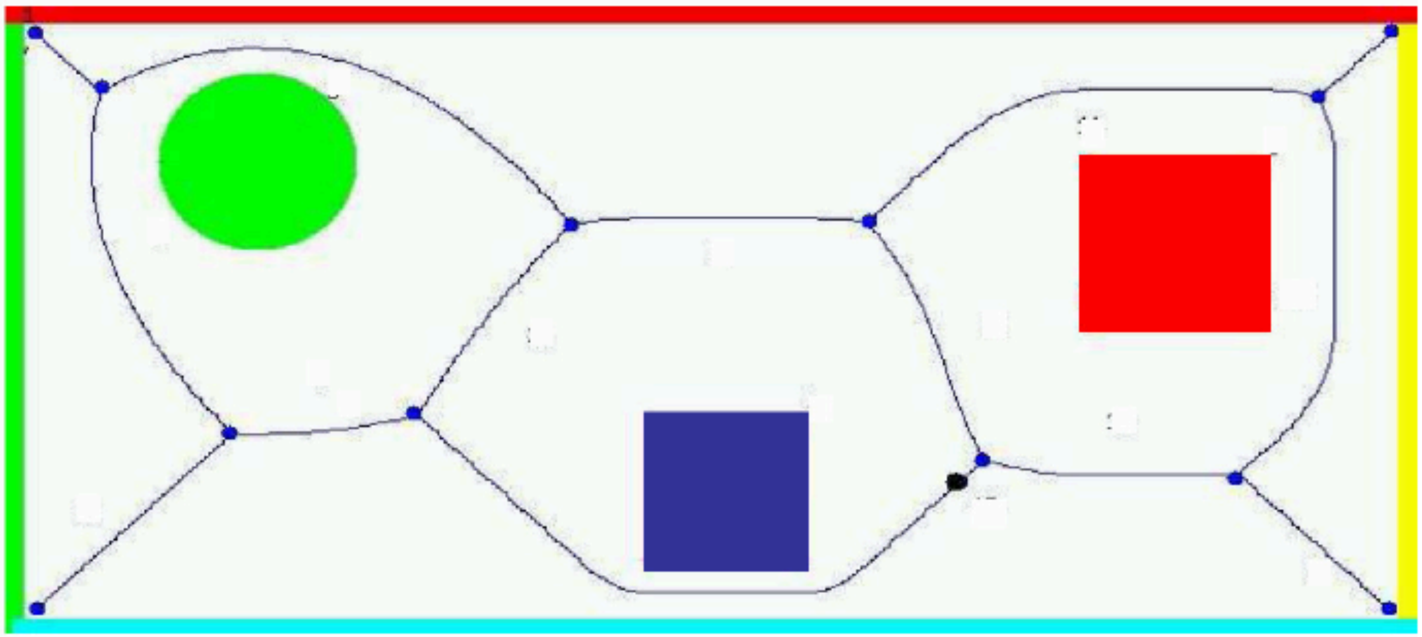
Voronoi Diagrams



- Complexity (in the plane):
- O(*N* log *N*) time
- O(*N*) space

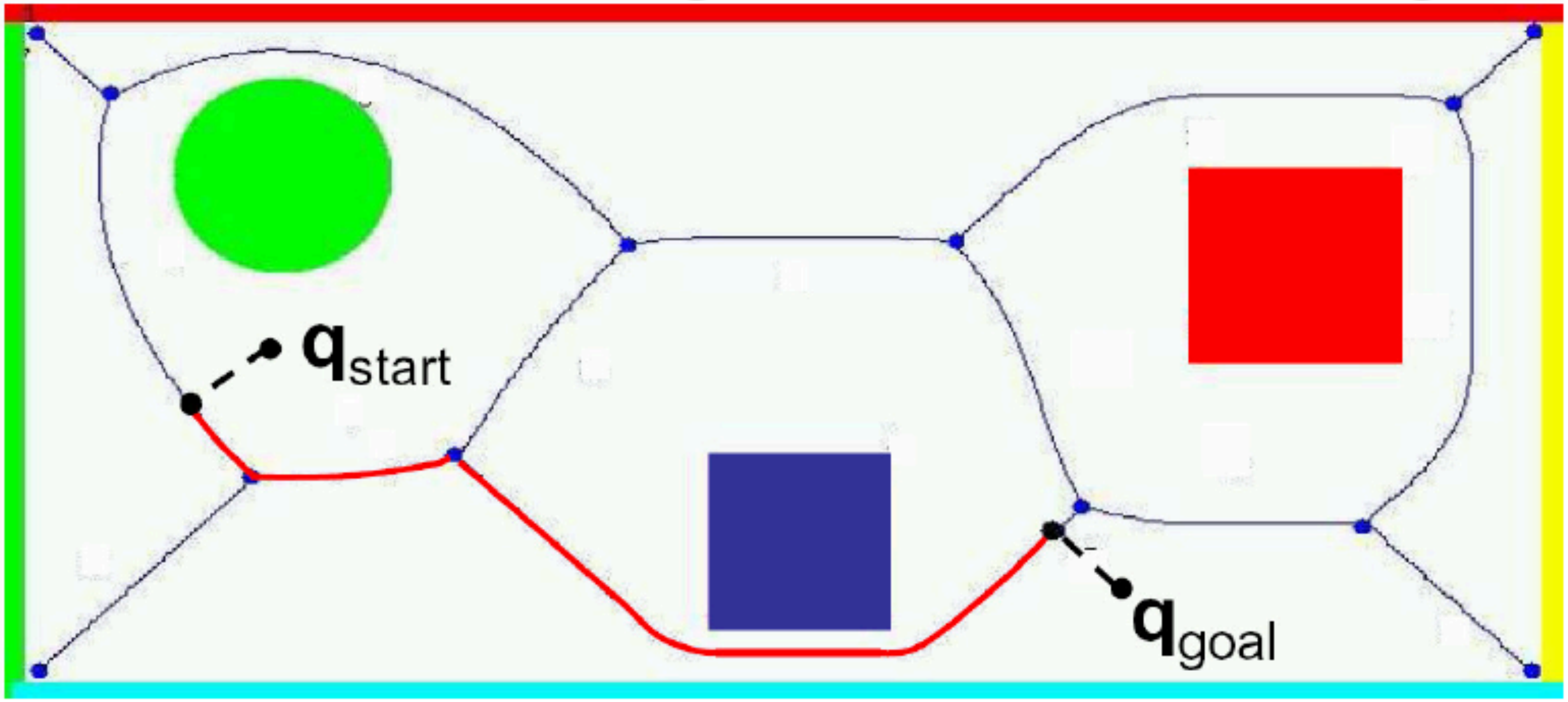
(See for example http://www.cs.cornell.edu/Info/People/chew/Delaunay.html for an interactive demo)

Voronoi Diagrams (Polygons)



- Key property: The points on the edges of the Voronoi diagram are the *furthest* from the obstacles
- Idea: Construct a path between $\mathbf{q}_{start}$ and $\mathbf{q}_{goal}$ by following edges on the Voronoi diagram
- (Use the Voronoi diagram as a roadmap graph instead of the visibility graph)

Voronoi Diagrams: Planning



- Find the point $\mathbf{q}^*_{start}$ of the Voronoi diagram closest to $\mathbf{q}_{start}$
- Find the point $\mathbf{q}^*_{goal}$ of the Voronoi diagram closest to $\mathbf{q}_{goal}$
- Compute shortest path from $\mathbf{q}^*_{start}$ to $\mathbf{q}^*_{goal}$ on the Voronoi diagram

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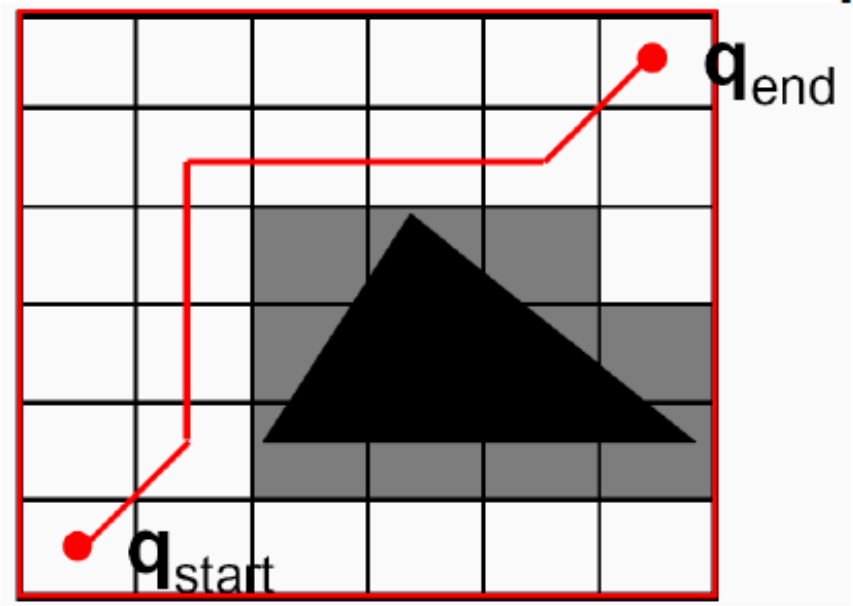
Voronoi: Weaknesses

- Difficult to compute in higher dimensions or nonpolygonal worlds
- Approximate algorithms exist
- Use of Voronoi is not necessarily the best heuristic ("stay away from obstacles") Can lead to paths that are much too conservative
- Can be unstable → Small changes in obstacle configuration can lead to large changes in the diagram

Approximate Cell Decomposition



- Define a discrete grid in C-Space
- Mark any cell of the grid that intersects $\mathcal{C}_{obs}$ as blocked
- Find path through remaining cells by using (for example) A* (e.g., use Euclidean distance as heuristic)
- Cannot be *complete* as described so far. Why?

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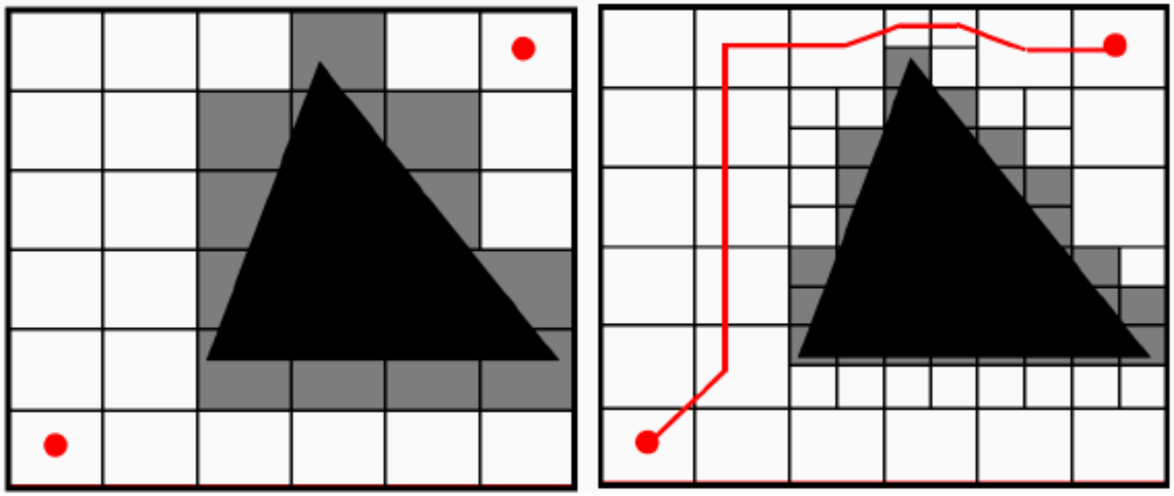
Approximate Cell Decomposition



- Cannot find a path in this case even though one exists
- Solution:
- Distinguish between
  - Cells that are entirely contained in $\mathcal{C}_{obs}$ (*FULL*) and
  - Cells that partially intersect $\mathcal{C}_{obs}$ (*MIXED*)
- Try to find a path using the current set of cells
- If no path found:
  - Subdivide the *MIXED* cells and try again with the new set of cells

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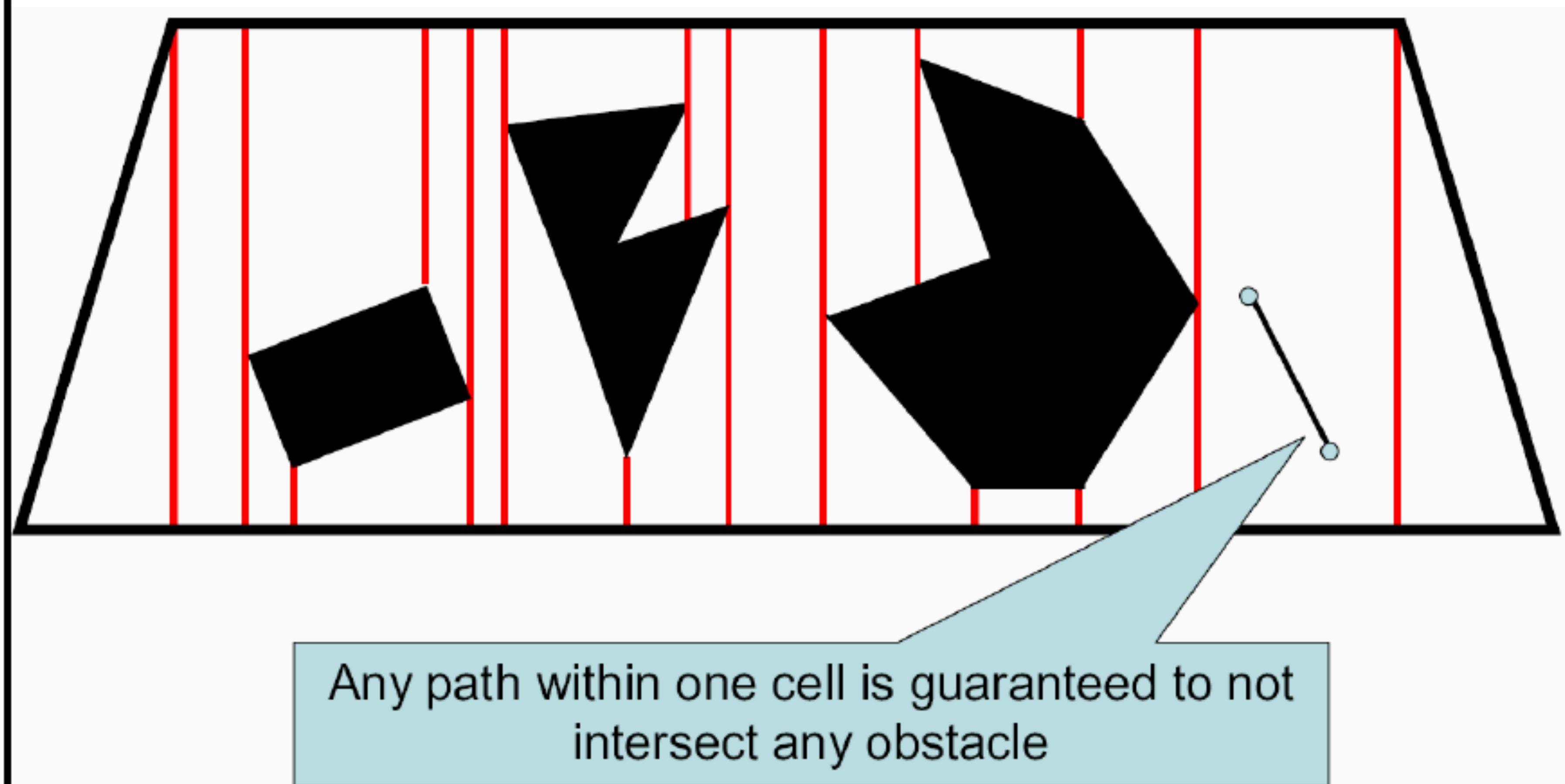
Approximate Cell Decomposition: Limitations

- Good:
  - Limited assumptions on obstacle configuration
  - Approach used in practice
  - Find obvious solutions quickly
- Bad:
  - No clear notion of optimality (“best” path)
  - Trade-off completeness/computation
  - Still difficult to use in high dimensions

Exact Cell Decomposition



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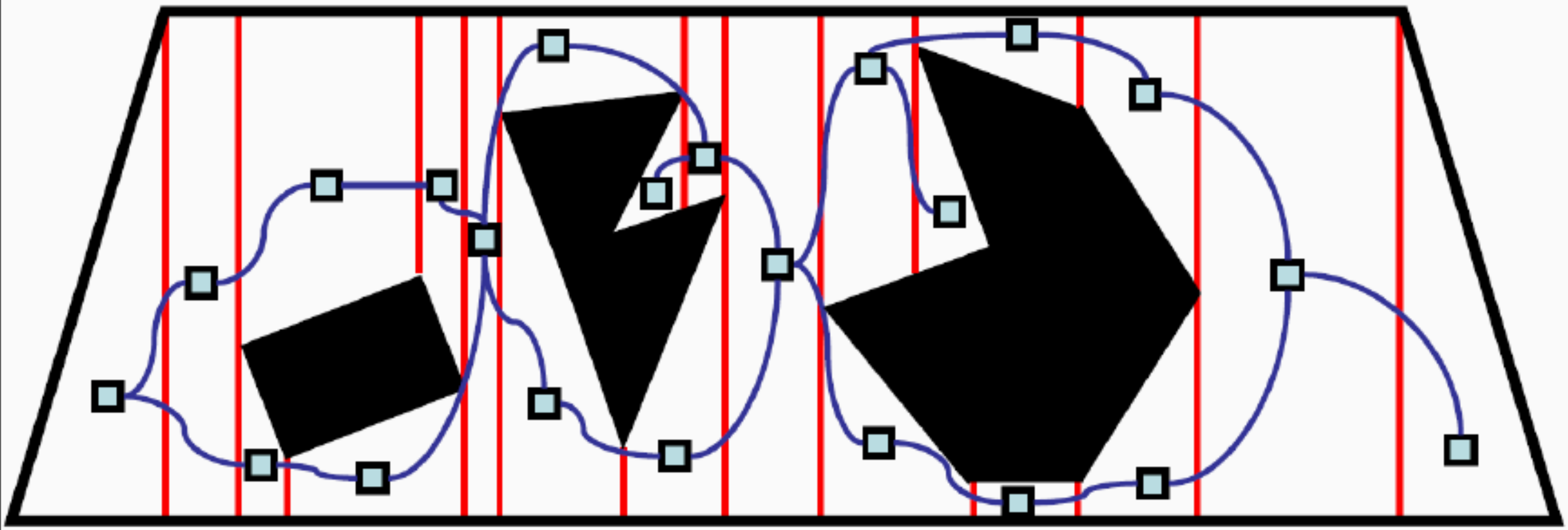
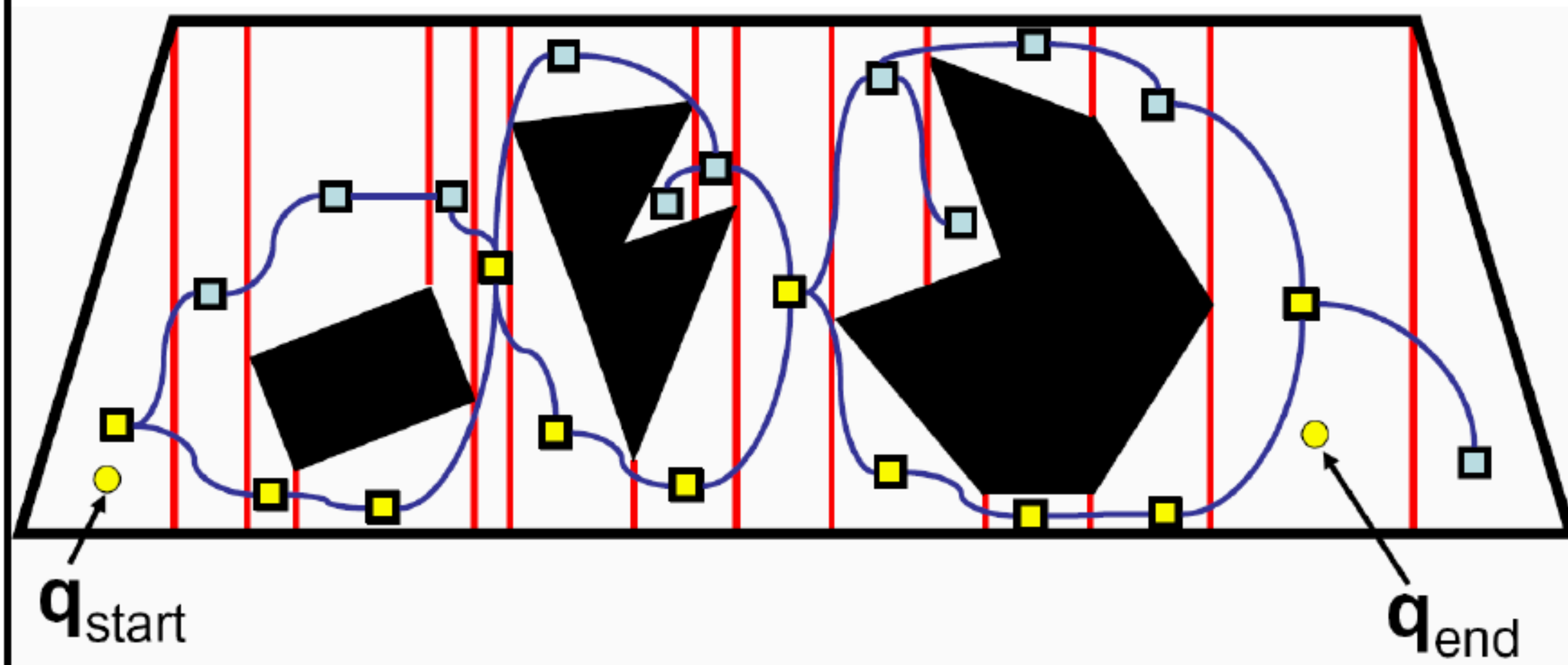
Exact Cell Decomposition



- The graph of cells defines a roadmap

Exact Cell Decomposition



- The graph can be used to find a path between any two configurations

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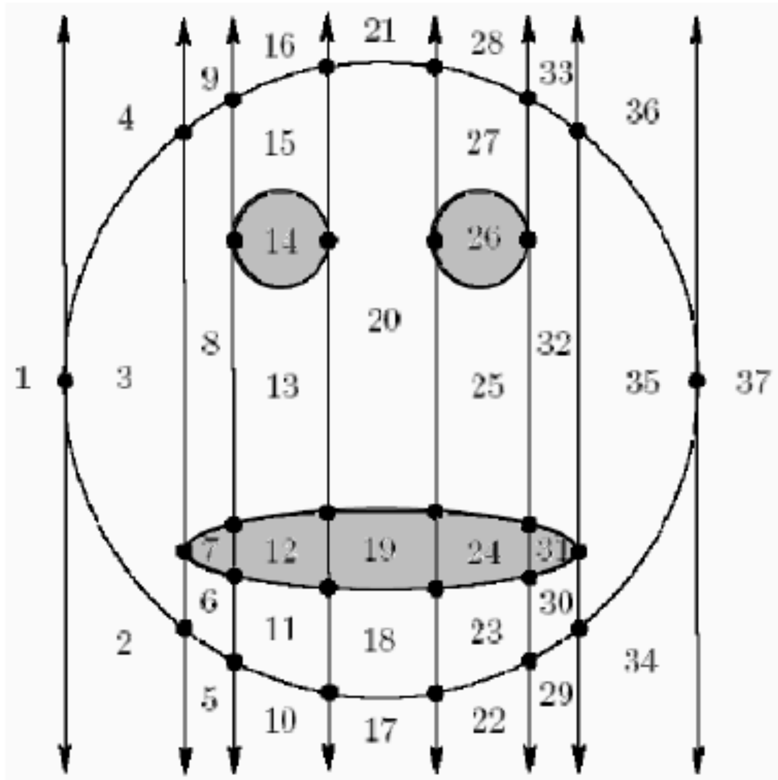
Exact Cell Decomposition



- A version of exact cell decomposition can be extended to higher dimensions and non-polygonal boundaries ("cylindrical cell decomposition")
- Provides exact solution → completeness
- Expensive and difficult to implement in higher dimensions

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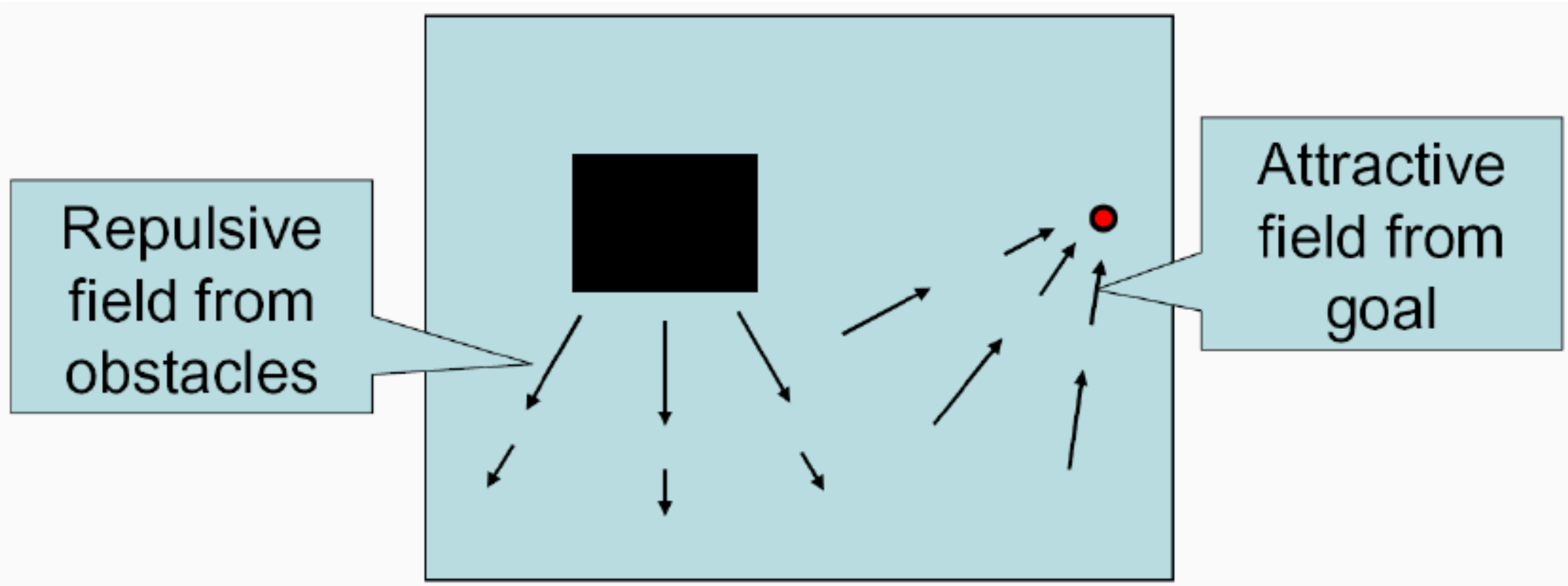
Potential Fields



- Stay away from obstacles: Imagine that the obstacles are made of a material that generate a *repulsive* field
- Move closer to the goal: Imagine that the goal location is a particle that generates an *attractive* field

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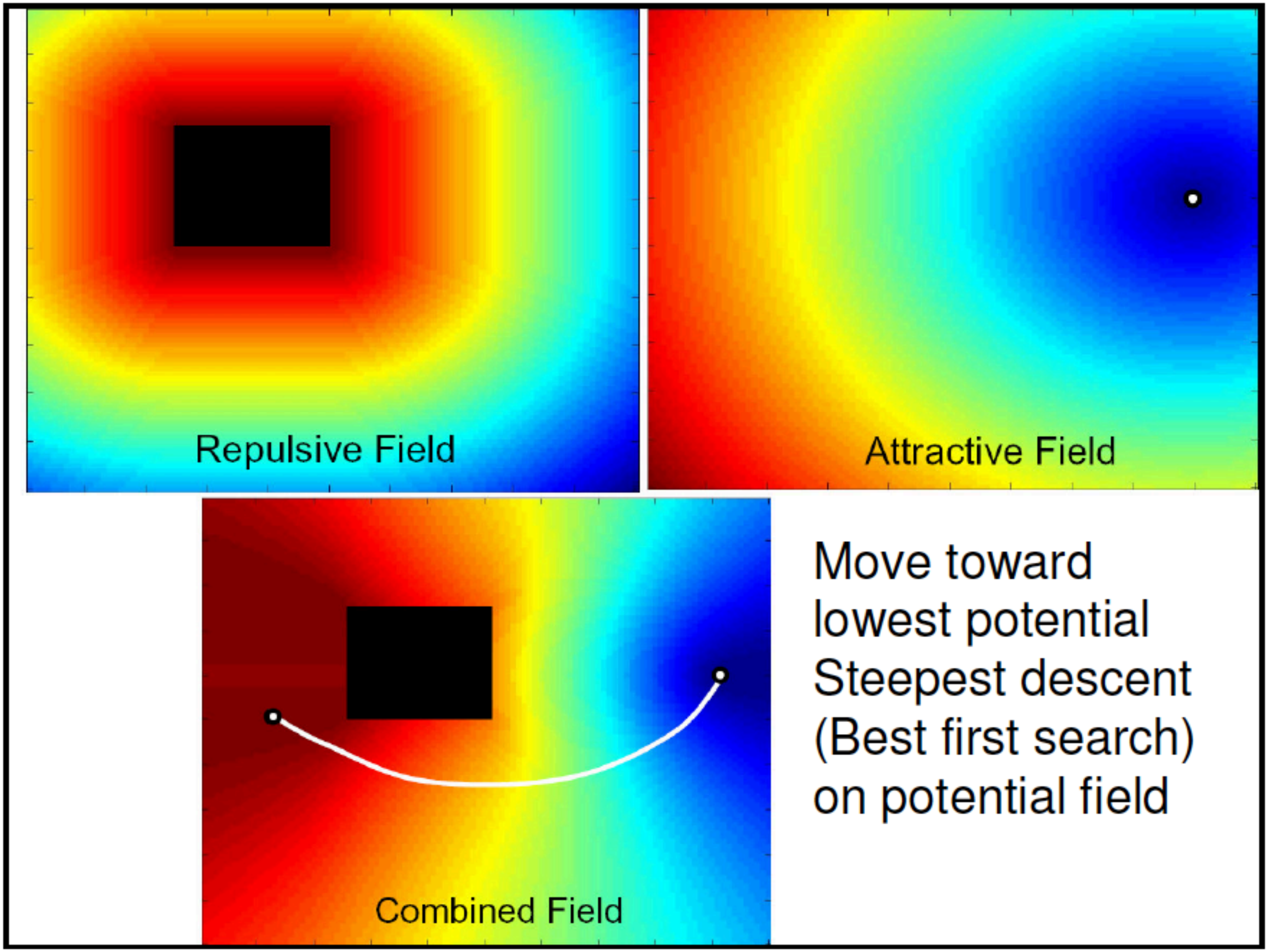

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$$U_g(\mathbf{q}) = d^2(\mathbf{q}, \mathbf{q}_{goal})$$

Distance to goal state

$$U_o(\mathbf{q}) = \frac{1}{d^2(\mathbf{q}, Obstacles)}$$

Distance to nearest obstacle point.
Note: Can be computed efficiently by using the *distance transform*

$$U(\mathbf{q}) = U_g(\mathbf{q}) + \lambda U_o(\mathbf{q})$$

λ controls how far we stay from the obstacles

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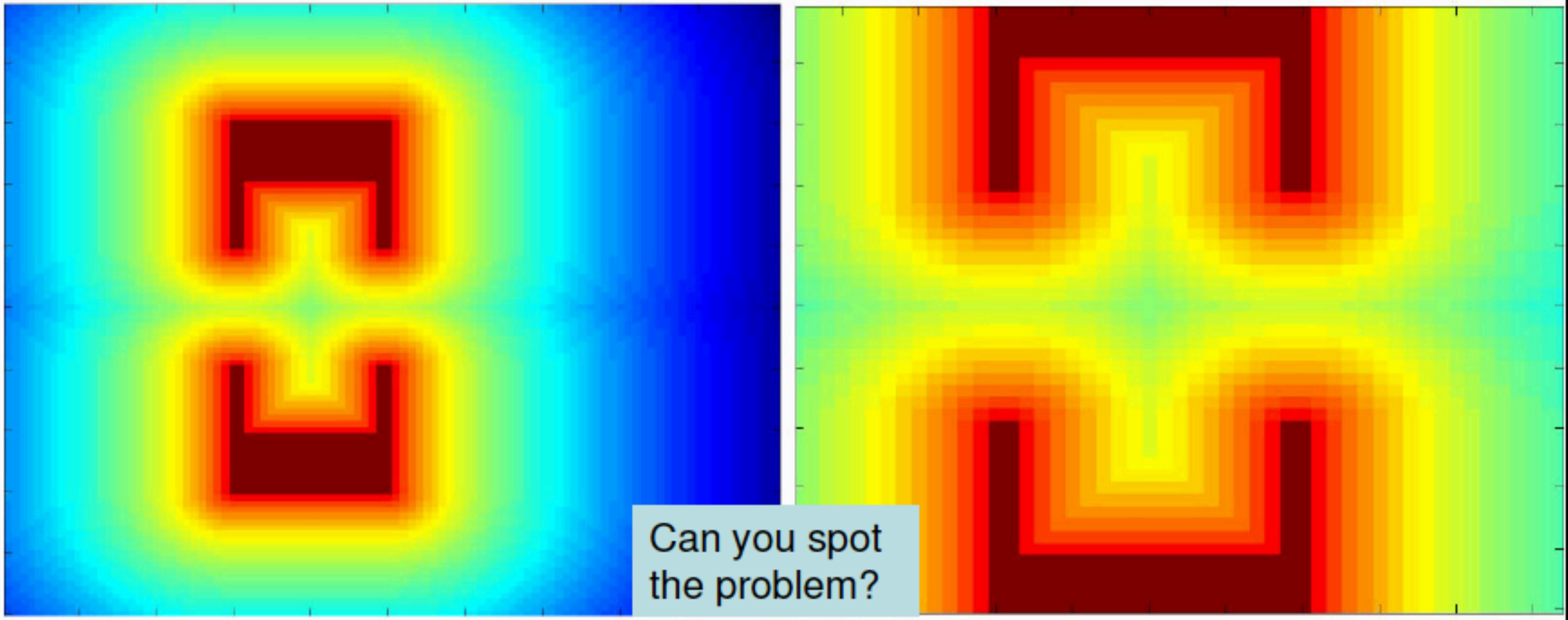
Potential Fields: Limitations



Potential field

Zoomed in view

- Completeness?
- Problems in higher dimensions

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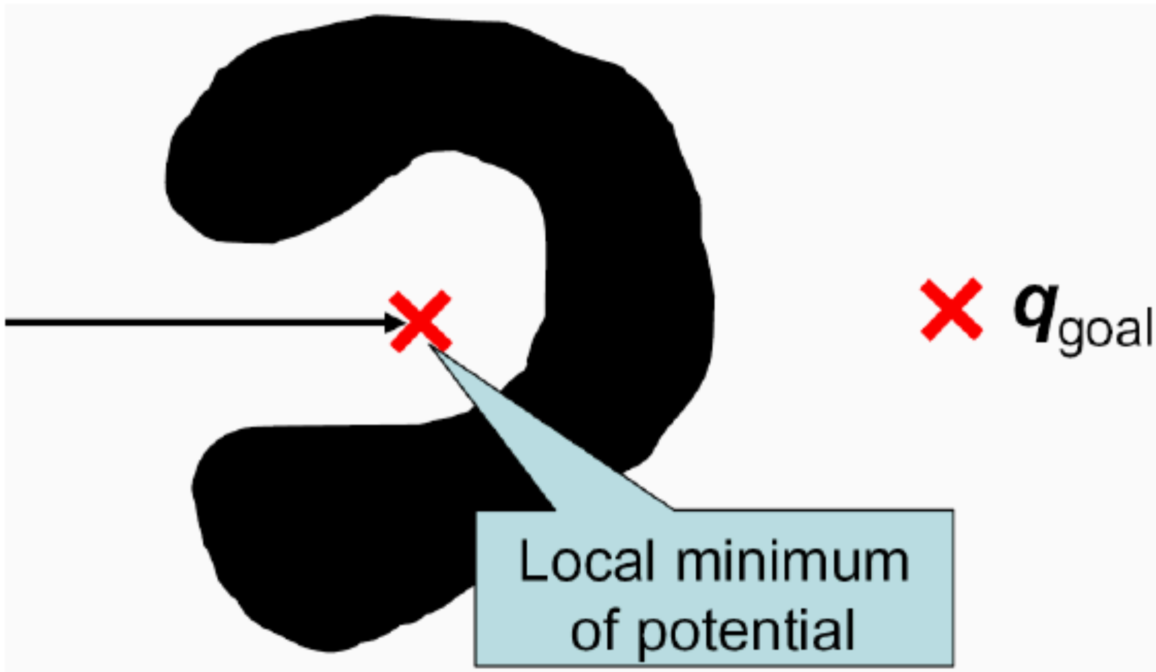
Local Minimum Problem



- Potential fields in general exhibit local minima
- Special case: Navigation function
  - $U(\boldsymbol{q}_{goal}) = 0$
  - For any $\boldsymbol{q}$ different from $\boldsymbol{q}_{goal}$, there exists a neighbor $\boldsymbol{q}'$ such that $U(\boldsymbol{q}') < U(\boldsymbol{q})$

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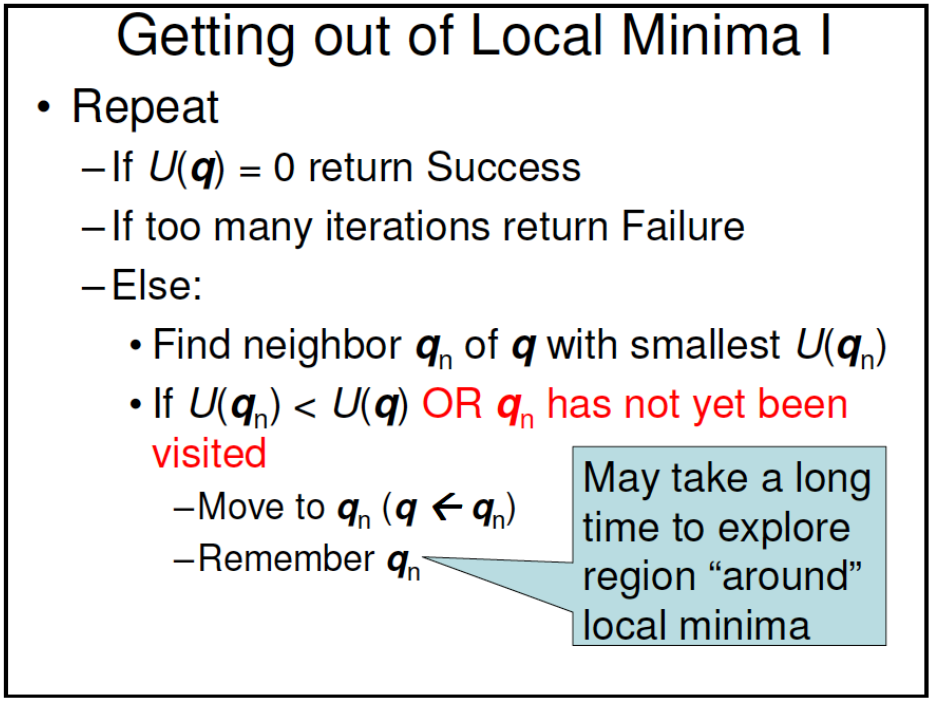
Getting out of Local Minima I

- Repeat
  - If $U(\boldsymbol{q}) = 0$ return Success
  - If too many iterations return Failure
  - Else:
    - Find neighbor $\boldsymbol{q}_n$ of $\boldsymbol{q}$ with smallest $U(\boldsymbol{q}_n)$
    - If $U(\boldsymbol{q}_n) < U(\boldsymbol{q})$ OR $\boldsymbol{q}_n$ has not yet been visited
      - Move to $\boldsymbol{q}_n$ ($\boldsymbol{q} \leftarrow \boldsymbol{q}_n$)
      - Remember $\boldsymbol{q}_n$

May take a long time to explore region "around" local minima



- Think of this the following way:
  - impose a grid
  - do depth first search on the potential
- Idea:
  - other kinds of search
  - randomization should help a lot
- Concern:
  - what if q has lots of neighbors?

Getting out of Local Minima II

- Repeat
  - If $U(\boldsymbol{q}) = 0$ return Success
  - If too many iterations return Failure
  - Else:
    - Find neighbor $\boldsymbol{q}_n$ of $\boldsymbol{q}$ with smallest $U(\boldsymbol{q}_n)$
    - If $U(\boldsymbol{q}_n) < U(\boldsymbol{q})$
      - Move to $\boldsymbol{q}_n$ ($\boldsymbol{q} \leftarrow \boldsymbol{q}_n$)
    - Else
      - Take a random walk for T steps starting at $\boldsymbol{q}_n$
      - Set $\boldsymbol{q}$ to the configuration reached at the end of the random walk

Similar to stochastic search and simulated annealing: We escape local minima faster

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Getting out of Local Minima II

- Repeat
  - If $U(\boldsymbol{q}) = 0$ return Success
  - If too many iterations return Failure
  - Else:
    - Find neighbor $\boldsymbol{q}_n$ of $\boldsymbol{q}$ with smallest $U(\boldsymbol{q}_n)$
    - If $U(\boldsymbol{q}_n) < U(\boldsymbol{q})$
      - Move to $\boldsymbol{q}_n$ ($\boldsymbol{q} \leftarrow \boldsymbol{q}_n$)
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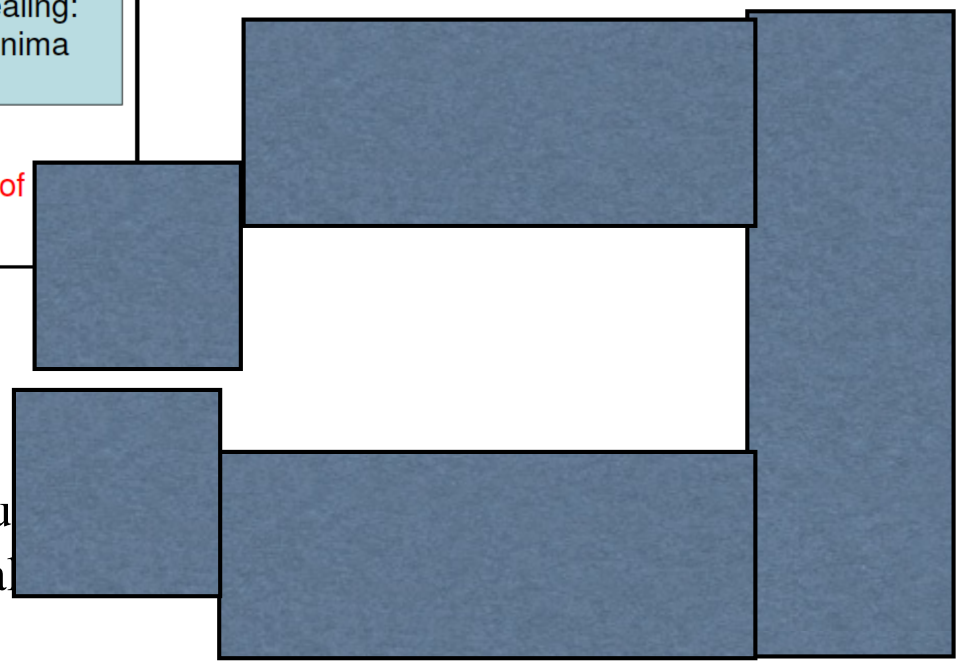

- Intuition:
  - random walk should get you
  - then slide down the potential
- Concern:
  - what if dimension is high?
    - random walk may not get out of local minima efficiently