Dynamics and MDP's

We have seen one optimization prob solved by D.P. (chain graph MRF's; forest MRF's)

- But there are many that fall to the same recipe
- E.G. LQR (linear quadratic regulator)
  - discrete time
  - continuous state.

$$x_{k+1} = A \underset{\uparrow\ \text{state}}{x_k} + B \underset{\uparrow\ \text{control}}{u_k}$$

Cost:

$$= x_N^T M x_N + \sum_{i=0}^{N-1} \left[x_i^T Q x_i + u_i^T R u_i + 2u_i^T N x_i \right]$$

cost of each state + cost of control

Consider LAST step.

$$x_N = A x_{N-1} + B u_{N-1}$$

$$\begin{aligned} \text{cost}(u_{N-1}) = \; & x_{N-1}^T A^T M A x_{N-1} \\ & + 2 x_{N-1}^T A^T M B u_{N-1} \\ & + u_{N-1}^T B^T M B u_{N-1} \\ & + 2 u_{N-1}^T N x_{N-1} + u_{N-1}^T R u_{N-1} \\ & + x_{N-1}^T Q x_{N-1} \\ & + \text{terms in other stuff} \end{aligned}$$

$\therefore$ best control:

$$\left(R + B^T M B\right) u_{N-1} = -\left[B^T M A + N\right] x_{N-1}$$

so $\quad u_{N-1} = -F x_{N-1}$

(Notice DP here!)

Now we set up an induction

$$x_N = A x_{N-1} - F x_{N-1}$$

so

cost $(x_{N-1} \ldots)$

$$= x_{N-1}^T \begin{bmatrix} A^T M A & - 2 A^T M B F \\ & + F^T B^T M B F \\ & - 2 F^T N \\ & + Q \\ & + F^T R F \end{bmatrix} x_{N-1}$$

$$+ \sum_{i=0}^{N-2} (\text{terms})$$

this is

$\cancel{A^T M A - (B^T M B + R)}$

$$\begin{bmatrix} A^T M A - [B^T M A + N]^T [R + B^T M B]^{-1} [B^T M A + N] \\ + Q \end{bmatrix}$$

Which yields u_{N-2}

etc!