Inequality constraints

Case 1: Boxes $l_i \le x_i \le u_i$

- first case: f is a quadratic form

$$\min_x \quad f(x) = \frac{x' G x}{2} + x'c$$

$$\text{st} \qquad l \le x \le u$$

- now if we are at x^k, the descent direction is $Gx + c$

- The ray in this direction is easy to project onto the box.

- Define Cauchy point to be first local minimizer of resulting piecewise quadratic function on ray.

Now, at this cauchy point x^{k^c},
some constraints are active
$i \in A$ ← active set. active

- produce approx soln to

$$S(x) = \frac{x' G x}{2} + c' x$$

$$\text{st} \quad x_i = x_i^c \qquad i \in A$$

$$l_i \le x_i \le u_i \qquad i \notin A$$

- We don't need an EXACT soln (which would be hard)

- require only a "soln" st $f(x^{k^s}) \le f(x^{k^c})$

- this is our new estimate.

1) method behaves well

2) Quite useful for DUAL problems

eg primal: $\min \quad x'\frac{G}{2}x + b'x$

$\quad \text{st} \quad Ax \geq b$

gives

$$\max_{\lambda} \quad \text{dual}(\lambda)$$

$$\text{st} \quad \lambda \geq 0$$

- Getting an approximate soln
- Consider

$$\min \quad \frac{x' G x}{2} + b' x$$

$$\text{st} \quad Ax = b$$

we could write $x = Zw + h$

↑ null space of A

and get

$$\min \quad w' H w + k' w$$

with no constraints.

- We would not usually use this form because Z is hard to get
- But if we did, could solve using Conj. grad.

- Notice, for ~~bound~~ constraints

$$x_i = x_i^c$$

Z takes a really simple form.

IDEA : C.G. on resulting system

- What about inequality constraints?

Two strategies:

- Stop C.G. iteration when we hit constraint.

- get C.G. result, then project

- This approach extends to

$$\min_{x} \quad f(x)$$

$$\text{st} \quad l \leq x \leq u.$$

- What about

$$\min_{x} \quad f(x)$$

$$\text{st} \quad Ax \leq b$$

- Strategy: slack vars ~~meet~~ have box constraints

hence

$$\min_{x} \quad f(x)$$

$$\text{st} \quad Ax - b + s = 0$$

$$s \geq 0$$

Concern:

- we may significantly increase the number of variables!

Interior point methods

we have

$$\min \quad f(x)$$

$$\text{st} \quad c_e(x) = 0$$

$$c_i(x) - s = 0$$

$$s \geq 0$$

KKT give

Jacobians

$$\nabla f(x) - J_E(x) \cdot y - J_I(x)\, z = 0$$

Lagrange mults

this is diag(s)

$$S z - \mu e = 0$$

$(1, \ldots 1)'$

0 for KKT above

μ for $s \geq \mu$

$$c_E(x) = 0 \qquad c_I(x) - s = 0$$

$$s \geq 0 \qquad z \geq 0$$

IDEA:

- work with $s \geq \mu$
- make μ smaller.
- should give path to solution

Another view (log penalty)

$$\min \quad f(x) - \mu \sum_{i=1}^{m} \log s_i$$

$$\text{st} \quad c_E(x) = 0$$

$$c_I(x) - s = 0$$

$\mu > 0$, - notice $-\log s_i$ gets huge as $s_i \to 0$

- this is called a penalty barrier.

IDEA - solve for μ
- make μ smaller
- path to soln.