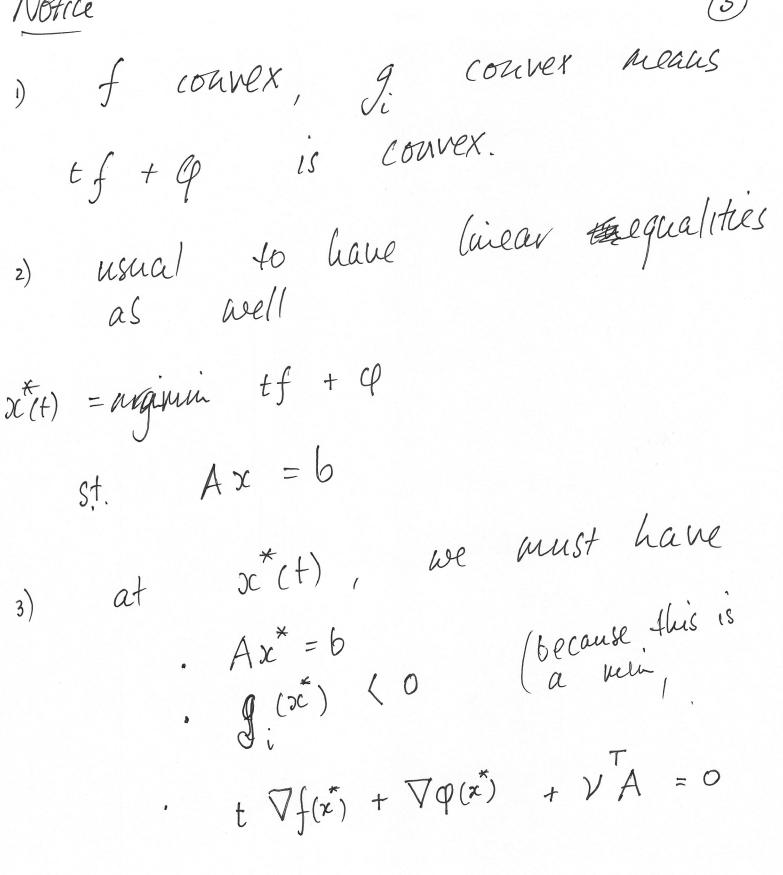
Interior Point methods 0 problem: min f St gox/x 0 (assume fliere à some DC St g. (De) KO, HL) avoid the boundary strategy: we wish to boundary may take large because progress on the be slow. It's have to constraints Steps, because the active overstep to might change. If we , projecting produce au infeasible pt. fough. back could be barmer Introduce a penalty $= -\sum_{i=1}^{r} \left(\frac{1}{2} \right) \log \left(-\frac{1}{2} \right)$ $\phi(x)$

As long as $g_i(x) < 0$, this is finite. (2) at $g_i(x) = 0$, the os for $g_i(x)$ wery negative (i.e. for from $for \quad g_i(x)$ wery negative (i.e. for from $foun \; Jarry$) is small. now $\min f(x) + \frac{1}{t} \cdot \varphi(x)$ (multiply through by t) So: μ in $t f(x) + \phi(x)$. write p(t) = argmin tf(x) + cp(x)This is called the <u>central path</u> large t : <u>x</u>(c) way in interior small t : <u>could be at</u> boundary

Notice



Which gives $M = \frac{1}{\sum_{i=1}^{m} \frac{1}{g(x^*)}} \sqrt{\frac{4}{2}} + \sqrt{4} = 0$ $E \nabla f(x^*) + \sum_{i=1}^{m} \frac{1}{g(x^*)} \sqrt{\frac{1}{2}} \sqrt{\frac{1}{2}}$ 4) there are λ^* , $\nu^* \in 2nal$ featible cornesponding to ∞^* write $\lambda_i^* = \frac{1}{t g(x^*)}$ Iluis is t ve. Huis rewrite optimality as flien

 $\nabla f + \sum_{i}^{*} \nabla g_{i}(x^{*}) + \nu^{*} A = 0$ Iluis is condition for a Lagrangian but So oct the building fes $+ \mathcal{V}^{\star T}(A \times -b)$ $f + \sum_{i} \hat{\lambda}_{i} g_{i}$ for λ^* , v^* feasible. = λ, ν^* is λual because we have is finite This is important, $g(\lambda, v)$ that the Inal at λ, ν^*

evaluate the that at h, v" We can $g(\lambda^*, \gamma^*) = \inf_{\mathcal{V}} \mathcal{I}(\mathcal{U}, \lambda^*, \gamma^*)$ $= \mathcal{L}(x, \lambda, y^{*})$ $= f + \sum_{i} \left[\frac{-1}{t g(x^*)} \right] g_i(x^*) + V^* \left[A x^* - b \right]$ -m/t = f(x) - m/ton Sola a lower bound This is upper bound So its trapped in a range [f(2e) - my, f(2e)] $f(x^{*})$



we have $\min t f(x) + \phi(x)$ St Ax=b KKT : Ax = b $g_i(x) \leq 0$ λ ; λ o $+\sum_{i}\lambda_{i}\nabla\phi_{i}+\nu^{T}A=0$ ∇f $-\lambda_i g_i(x) = 1/t$ A from Zetu & 2*

Notice that these look like KKT for original problem, with a change in the complementarity coud. start with feasible x, t°ro, m >1 z ro Alg: $x^* = argnin ff + \varphi$ sf A x = b (AIAI7)(ALM?) quit else t=mt $x = x^*$ HMCZ

8

What about SDP?

 $\begin{aligned} \varphi(\mathbf{e}\mathbf{x}) &= -\log\left[\det(\mathbf{e}\mathbf{x})\right] \\ \psi(\mathbf{e}\mathbf{x}) &= -\log\left[\det(\mathbf{e}\mathbf{x})\right] \\ \text{is the solution of the start of t$ $\frac{\partial}{\partial x_{ij}} \left(\log \det X \right) = \frac{1}{\det(X)} \left[\operatorname{coeff} \delta x_{ij} \text{ in } \partial et \right]$ graduents: written. $\frac{2}{3}$ (log det X) = X⁻¹ Sometimes

Primal- Inal methods flat a path following method hecall KKT Solve 2 $A \propto = b$ g(x) 50 $\lambda_i > 0$ $\nabla f + \sum_{i} \lambda_i \nabla g + \mathcal{Y} A = 0$ $-\lambda_i f_i(x) = 1/t$ But we solved for x* for given t, then recovered instead view KKT - we could of equis in 2, 2, 20 ces a System solve, make t for given t, Smaller, etc. bigger

to solve? they're nonlinear, even it they're nonlinear, even it inequality constraints are how a) lucar $-\lambda_i f_i(x) = 1/t$ Nasty complementainty cond. 6). see this as root finding. b). See Newton. G(u) = 0fint $G(u^{(n)} + \Delta u) \approx G(u^{(n)}) + J_{g} \Delta u$ (n). then $J_{G}\Delta u = -G.$ 50

· - /

Our system

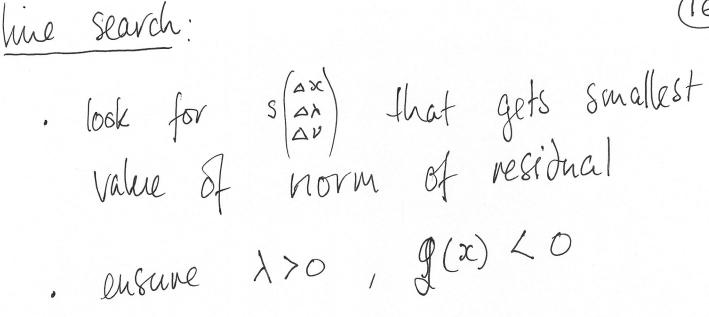
A dual residual If this is 0, 2, 1 ave df centering residual $\nabla f + \lambda^T J_g + \nu^T A$ $-\operatorname{diag}(\lambda).g = -\frac{1}{t}.1$ e primal résidual Aoc-b 4. centering residual expresses whether point is on central path point is residual is or feasible primal residual 1d Гс Гр

(12)

Newton's - rel Hs + Zx; Hg; - r. - diag(1).Jg - (p A But how good is the point? we don't necessarily have either prind feasible of the Inal feasible - so we can't get Fuality Instead, work with surrogate X,V yap Juality gap:

· assume x is primal feasible -then primal has value f(x). then x, v are dual feasible, And we're at value Jual has value $f(x) + \lambda g$ So gap is primal - Inal T $\chi = -\lambda g$ (Notice here where assuming g Xo;) XXO notice also that this is related to complementarity on $-\lambda_i g_i = \frac{1}{t}$ $or t = \tilde{\eta}$ $\int_0 \hat{\eta} = \frac{M}{t}$

Algorifhm: Start 4 $\left(q(z) < 0\right)$ with flasible x $\lambda > O$ h pr > 1 2, >0 2 70 Herate t = M Mget line search dir beg hueger algon System 6 find s> 0 above (me search; 0 $\begin{array}{c} \chi \\ \rightarrow \chi \\ + S \\ \lambda \end{array}$ х Л Prink Z feas 1 d I Trink Z feas n n KZ. V until



find surgest s=sthat gives 1>0now search back by $s^{(n+1)} = \beta s^{(n)} \quad 0 < \beta < 1$ strategy looking for sufficient Improvement in residual