

③-①

Duals of linear programs

1) Assume we have a linear program

$$\min \quad -w^T f$$

$$\text{s.t.} \quad A_e f = 0$$

$$f \geq 0$$

$$c - f \geq 0$$

(This isn't the standard form - just convenient for flow)

Its Lagrangian is

$$\mathcal{L}(f, \lambda_e, \lambda_i^1, \lambda_i^2)$$

$$= -w^T f + \lambda_e^T (A_e f) - \lambda_i^{1T} f - \lambda_i^{2T} (c - f)$$

(3-2)

we want the Lagrange dual.

$$\inf_{\mathbf{f}} \mathcal{L}(\mathbf{f}, \lambda_e, \lambda_i^1, \lambda_i^2) \\ = \begin{cases} -\lambda_i^{2T} \mathbf{c} & \text{if } [-\omega^T + \lambda_e^T \mathbf{A}_e - \lambda_i^{1T} + \lambda_i^{2T}] = 0 \\ -\infty & \text{otherwise.} \end{cases}$$

Now the dual problem is to max this

i.e.

$$\begin{aligned} \max & \quad -\lambda_i^{2T} \mathbf{c} \\ \text{s.t.} & \quad -\omega^T + \lambda_e^T \mathbf{A}_e - \lambda_i^{1T} + \lambda_i^{2T} = 0 \\ & \quad \lambda_i^1 \geq 0 \\ & \quad \lambda_i^2 \geq 0 \end{aligned}$$

We can clean this up a bit,

③-③

$$\max - \lambda_i^2 C$$

$$\text{s.t. } A^T \lambda + \lambda_i^2 \geq w^T$$

$$\lambda_i^2 \geq 0$$

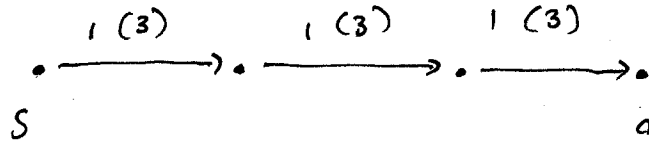
9

Now, notice:

• in an augmenting path, we always had a flow

- is this a cut? not necessarily

- eg.



- flow that isn't a cut, BECAUSE there is an augmenting path.

- cf. discussion of aug paths.

- a flow is ONLY a cut if there is no augmenting path.
i.e. if it's maximal.

In push-relabel, we have manipulating
CUTS (by them, above), and ONLY
 had a flow at optimality.

~~These two are dual~~

Max-flow, min-cut are DUAL

MAX FLOW

$$\min -w^T f$$

$$\text{s.t. } A_R f = 0$$

$$f \geq 0$$

$$C - f \geq 0$$

MIN CUT

$$\max -\lambda_i^{2T} C$$

$$\text{s.t. } -w + A_R^T \lambda_e - \lambda_i^1 + \lambda_i^2 = 0$$

$$\lambda_i^1 \geq 0$$

$$\lambda_i^2 \geq 0$$

