Inequality constraints - some special cases (1)

- Box constraints

$\underline{l} \leq \underline{x} \leq \underline{u}$ vector of bounds.

upper

lower

- AND no equality constraints.

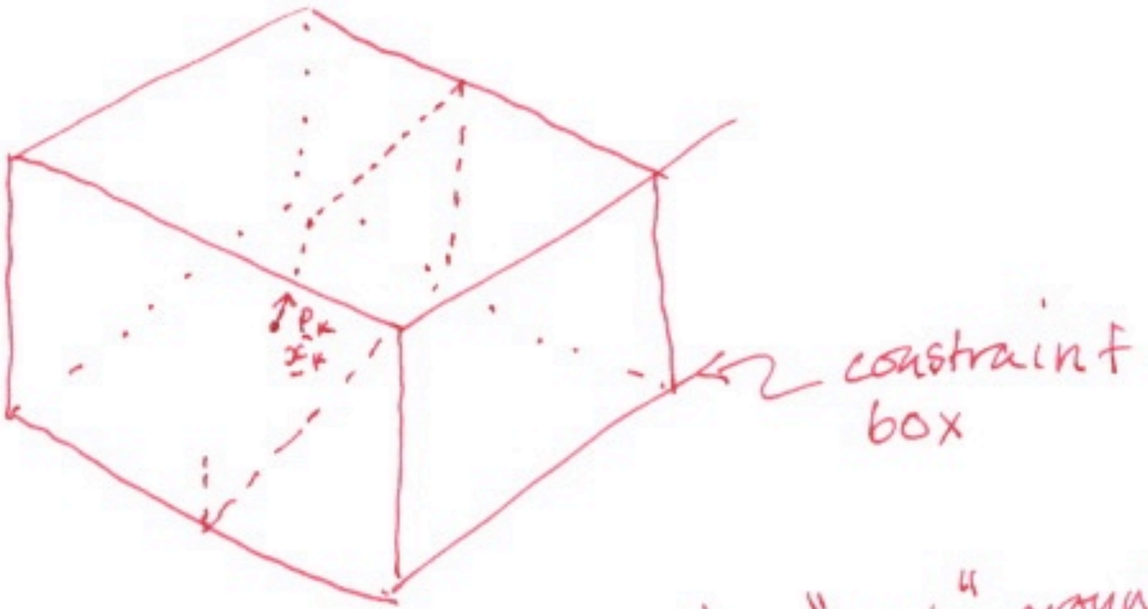
- Function is

min $\quad \underline{x}^T \frac{G}{2} \underline{x} + v^T \underline{x}$

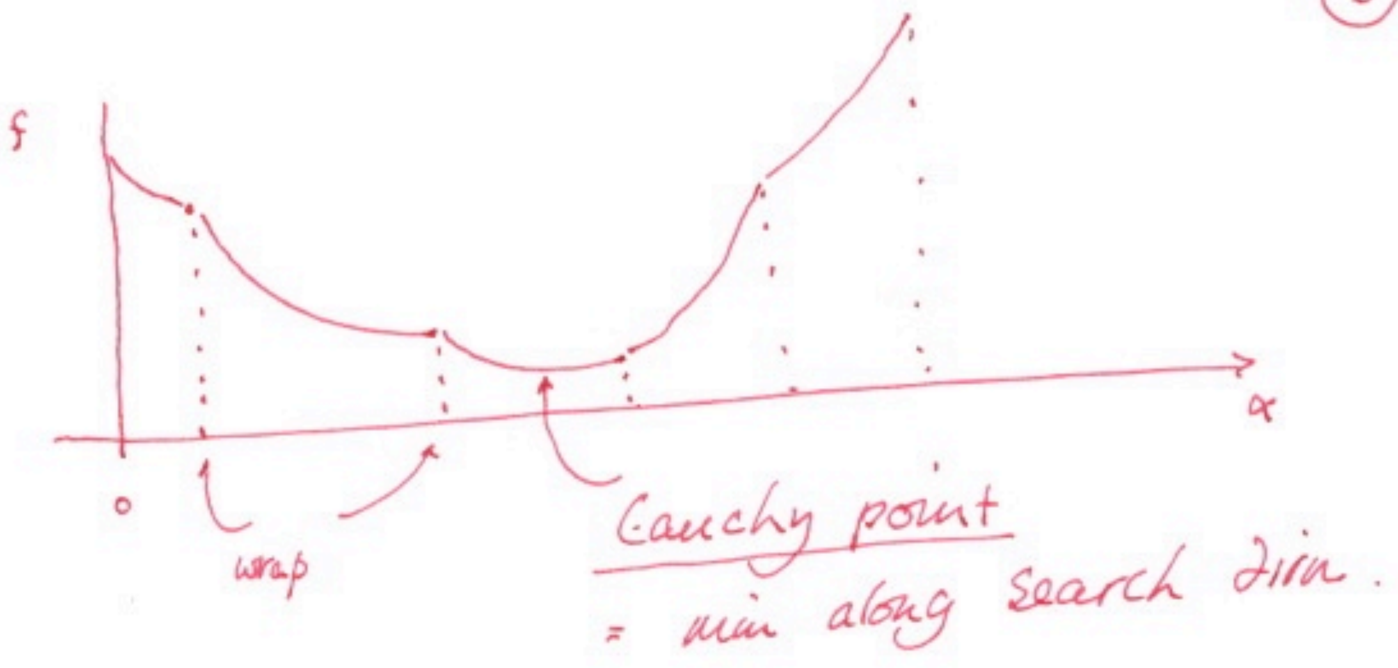
and: $\quad G$ is P.D. ← important: (Why?)

- we are at $\underline{x}_k$ and want to take a step
  - dir'n is $\underline{p}_k$ (which we might have gotten in a variety of ways - Newton, Q.N., Grad, etc.)

Consider the search



- ie take ~~dirn~~ ray $\underline{x}_k + \alpha \underline{p}_k$ and "wrap" around box
- Now consider the objective function, restricted to this path, as a function of α
  - (a) continuous
  - (b) piecewise quadratic
  - (c) bounded below



· C.P. is relatively easy to find.

eg between α_i , α_{i+1} knots.

function has form

$$\frac{a}{2}\alpha^2 - b\alpha$$

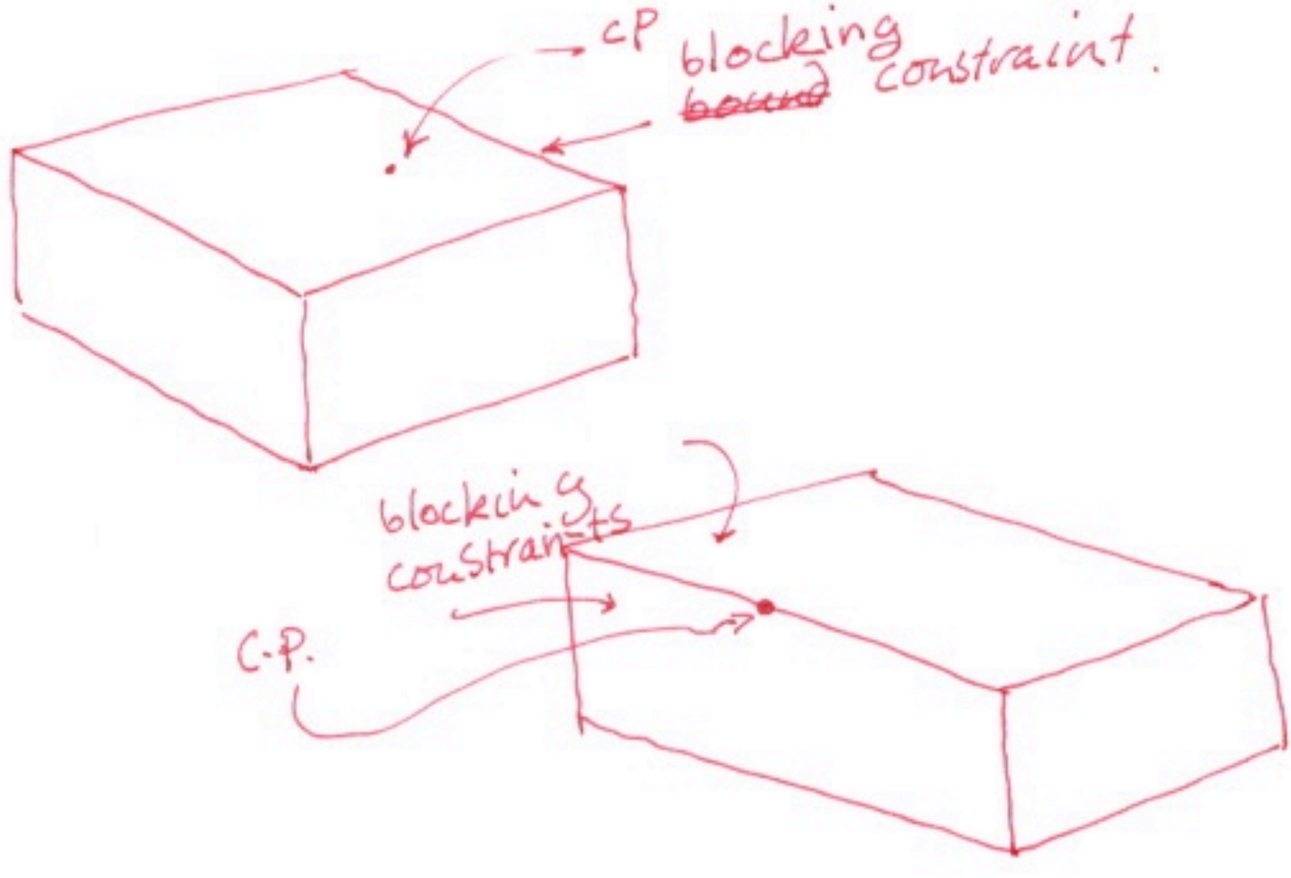
so
$$\alpha = \begin{cases} \alpha_i & \text{if } \frac{b}{a} < \alpha_i \\ \frac{b}{a} & \text{otherwise} \\ \alpha_{i+1} & \text{if } \frac{b}{a} > \alpha_{i+1} \end{cases}$$

and if $\alpha = \alpha_{i+1}$, we need to generate a new wrap.

At Cauchy point, we fix the blocking ~~bound~~ constraints, then search face

constraints that are currently active

eg



searching the face

- we can rewrite.

model at C.P. is

$$\frac{x^T G x}{2} + d^T x$$

$$x_i = x_i^c$$ the blocking constraints

$$l \leq x \leq u$$

- Now substitute for these blocking constraints
- write $$\underline{x} = \begin{pmatrix} 0 \\ \vdots \\ x_i \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \underline{v}_a \\ 0 \\ \underline{v}_b \end{pmatrix}$$ unknown

known values, from blocking constraints

this gets us

$$\min \quad v^T \frac{Q}{2} v + e^T v$$

$$\text{st} \quad \hat{\ell} \le v \le \hat{u}$$

leave out the bits csp to known values

and we have an initial v = C.P.

- Apply C.G., stopping when a constraint is violated.

Q1: why do I get a better v like this?

Q2: why not apply to

$$\boxed{\begin{array}{l} x^T \frac{Q}{2} x + b^T x \\ \text{st} \quad Ax = c. \end{array}}$$

?