Regret: and SGD

- at each step, we see a new objective function

- online optimization

 at time step t, alg sees $f_t : K \to R$ and proposes x_t

- how to score this?

$$\text{regret} = \sum_t f_t(x_t) - \min_{x^* \in K} \sum_t f_t(x^*)$$

time average of cost functions ~~at~~ chosen points.

you see all ~~~~ fn's in advance – find smallest average.

the reason this is helpful is we can prove bounds, which lead to new algs ②

- take g.d. and substitute

$$x_{t+1} \longleftarrow x_t - \eta \nabla f_t(x_t)$$

use current function

~~not~~

from proof of G-2, we have

$$f_t(x_t) + \Phi_{t+1} - \Phi_t \leq f_t(x^*) + \frac{1}{2}\eta G^2$$

(cause true for any convex).

Sum this

$$\sum_{t=1}^{T}\left(f_t(x_t) - f_t(x^*)\right) \leq \sum_{t=1}^{T}\left(\Phi_t - \Phi_{t+1}\right) + \frac{1}{2}\eta G^2$$

$$\leq \Phi_1 + \frac{1}{2}\eta T G^2$$

now plug in $T \geq \frac{\|x_1 - x^*\|^2 G^2}{\varepsilon^2}$

$\eta = \frac{\|x_1 - x^*\|}{G\sqrt{T}}$, do algebra. to

get.

$$\frac{1}{T}\sum_{t=0}^{T}\left(f_t(x_t) - f_t(x^*)\right) \leq \frac{\|x_1 - x^*\| G}{\sqrt{T}} \leq \varepsilon$$

regret.