Inequality constraints: Working set methods.

Notice from KKT that some inequality constraints are active at solution, blocking others aren't. Active constraints behave like equalities — others don't count.

IDEA:

- Search for active constraints while searching for point.

Strategy:

- Start with feasible point.
- Improve, while keeping feasible.
- Maintain model of working set.

We will only consider linear constraints.
• assume we are at $x_k$.

• our model is

$$\min \quad x^T M x + d^T x$$

$$\text{st.} \quad a_i^T x + b_i = 0$$

all active constraints

• the we want

$$p \quad \text{st} \quad x_{k+1} = x_k + p$$

• so min

$$\min_p \quad \frac{x_{k+1}^T M x_{k+1}}{2} + d^T x_{k+1}$$

$$\text{st} \quad a_i^T x_{k+1} + b_i = 0$$

substitute, get

$$\min_p \quad p^T G p + e^T p$$

$$\text{st} \quad a_i^T p = 0$$

active constraints

Q: How do we solve this?
Notice that $x_k + \alpha \cdot p$ is feasible for all $\alpha$ and all working constraints.

Cases:

(A) $x_{k+1}$ is feasible for all constraints (easy check — they're linear)

$\rightarrow$ accept it.

(B) It isn't. — then there is some $\alpha \in [0, 1)$ such that $x_k + \alpha \cdot p$ is feasible (cause our working set is right — $x_k$ is feasible in W.S.
Case B:

consider a constraint NOT in w.s.

if \( a_i^T \rho > 0 \)
then \( a_i (x_k + \alpha\rho) > -a_i^T x_k \geq b \)

ANY +ve \( \alpha \) will work for this constraint.

if \( a_i^T \rho < 0 \) then

\[
\alpha_k \leq b_i - \frac{a_i^T x_k}{a_i^T \rho_k}
\]

Walk constraints to find smallest \( \alpha_k \).

constraints that have smallest \( \alpha_k \) are active/blocking/etc.

(\( \alpha_k \) could be zero if we're missing a constraint in working set.)
insert a blocking constraint into \( W_k \) to get \( W_k \)

- Meeting: Removing redundant constraints

Q: How do we know there is a constraint in working set that shouldn't be there?

A: Lagrange mult is \(-ve\).

Q: Where did LM come from?

A: recipe for quadratic form st. linear constraints
Overall story

- start $x_0$ feasible
- until finished
  - until $x_k$ is minimizer of QP over working set
  - Take a step, and adjust W.S.
  - Minimizer when $p = 0$
  - Any we hagrange nulls?
    - Yes: remove one from Ws
    - No: finished

Check:
- at finished, KKT are true.

Concern:
- How much work to adjust W.S.?
- Could this cycle?
- What if constraints aren't linear?