Segmentation to more than 2 classes

- The flow-cut equivalence no longer works.

- Assume we have r classes.
  - encode class at each pixel with a one-hot vector

label at ij $\quad \underline{l}_{ij} = \underbrace{[0 \; \cdots \; 1 \; \cdots \; 0]}$

r d vector
1 element $= 1$ others 0

- Notice

$$\underline{l}_{ij}^{T} 1 = 1$$

$$\underline{l}_{ij} \in \{0,1\}^{r}$$

we have some form of prob model per class, and use the same kind of smoothing.

So:

$\underline{\min}$ linear term in $[\underline{l}_{ij}]$ + Quad term in $[\underline{l}_{ij}]$

$\underline{st}$ $\quad \underline{l}_{ij}^T 1 = 1$

$\quad \underline{l}_{ij} \in \{0,1\}^r$

I: this is easily turned into a linear program

II: this linear program does not have tractable constraints
- we can't get a TUM problem out of it

III: solving problems of this kind is useful

I: turning into a linear program

(lose an index on labels)

$\underline{l}_a(i) = \{ i\text{'th component of } \underline{l}_a \}$

$q_{ab}(i,j) = \{ i,j\text{'th component of } \underline{q}_{ab} \}$

q_{ab} is an $r \times r$ table that represents the product $l_a \times l_b$

$$q_{ab}(i,j) = \begin{cases} 1 & \text{if } l_a(i) = 1 \text{ and } l_b(j) = 1 \\ 0 & \text{otherwise} \end{cases}$$

Note: $q_{ab}(i,j) \in \{0,1\}^{r \times r}$

$\underline{1}^T q_{ab} \underline{1} = 1$ (theres only one one in q_{ab})

Then

$$q_{ab}(i, \cdot) \leqslant \ell_a(i)$$

$$q_{ab}(\cdot, j) \leqslant \ell_b(j)$$

$$q_{ab}(i,j) \geqslant \ell_a(i) + \ell_b(j) - 1$$