

Dynamics and MDP's

We have seen one optimization prob solved by
d.p. (chain graph MRF's; forest MRF's)

but there are many that fall to the same
recipe

E.g. LQR (linear quadratic regulator)
- discrete time
- continuous state.

$$x_{k+1} = Ax_k + Bu_k$$

↑ ↑
state control

cost:

$$= x_N^T M x_N + \sum_{i=0}^{N-1} [x_i^T Q x_i + u_i^T R u_i]$$

cost of each state + cost of control

Consider LAST step.

$$x_N = Ax_{N-1} + Bu_{N-1}$$

$$\begin{aligned}
 \text{cost}(u_{N-1}) = & x_{N-1}^T A^T M A x_{N-1} \\
 & + 2x_{N-1}^T A^T M B u_{N-1} \\
 & + u_{N-1}^T B^T M B u_{N-1} \\
 & + 2u_{N-1}^T N x_{N-1} \\
 & + x_{N-1}^T Q x_{N-1} \\
 & + u_{N-1}^T R u_{N-1} \\
 & + \text{terms in other stuff}
 \end{aligned}$$

∴ best control :

$$(R + B^T M B) u_{N-1} = - [A^T B^T M A + N] x_{N-1}$$

so $u_{N-1} = -F x_{N-1}$

(notice DP here!)

Now we set up an induction

(3)

$$x_N = Ax_{N-1} - Fx_{N-1}$$

so

$$\text{cost} = (x_{N-1} \dots)$$

$$= x_{N-1}^T \left[\begin{array}{l} A^T M A \\ - 2 A^T M B F \\ + F^T B^T M B F \\ - 2 F^T N \\ + Q \\ + F^T R F \end{array} \right] x_{N-1}$$

$$+ \sum_{i=0}^{N-2} (\text{terms})$$

this is

$$\left[\begin{array}{l} \cancel{A^T M A} - \cancel{(B^T M B + R)} \\ A^T M A - [B^T M A + N]^T [R + B^T M B]^{-1} [B^T M A + N] \\ + Q \end{array} \right]$$

Which yields u_{N-2}

etc. !