Variational problems can be quite delicate

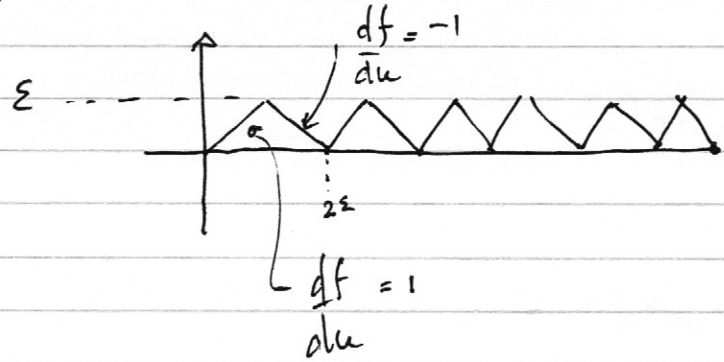
– a solution could not exist in reasonable function spaces.

example

$$\min_f \int_0^1 f(u)^2 + \left[\left(\frac{df}{du}\right)^2 - 1\right]^2 du = J[f]$$

s.t. $f(0) = 0$; $f(1) = 0$.

Soln looks like



<u>but</u> as ε gets smaller, we have that J gets smaller, too

→ <u>but there can't be a limit</u>

So <u>no solution</u>.

Failure of a Solution to exist occurs in practical problems

- Simple issues:

$$\operatorname*{argmin}_u \quad u^2 \qquad u \in (0, 1]$$

open.

(This doesn't happen all that often, but is worth keeping in mind)

- Harder issues.

$$\operatorname*{argmin}_f \int_0^1 \left[\frac{1}{\sqrt{1+(f')^2}} - \frac{1}{\sqrt{2}} \right]^2 dx$$

s.t. $f(0) = 0$

$f(1) = 0$

This sort of thing turns up in Shape from Shading problems rather often.

Notice I can get a min of the objective if $f'^2 = 1$.

$\rightarrow$ So, if $f \in C^\infty$ no solution (there can't be a C^∞ function st $f'^2 = 1$, $f(0) = 0$, $f(1) = 0$)

$\rightarrow$ if $f \in C^0$, too many solutions!

$\wedge$, $\wedge\wedge$, $\wedge\wedge\wedge$ etc.

~~Now assume that~~

In practice, we usually turn variational problems into continuous optimization problems by writing

$$f = \sum_i a_i \, g_i$$

basis functions

then solving for a_i

<u>BUT</u>

- bad stuff can happen if original problem is poorly set
- The <u>reasoning</u> comes in useful later

<u>Crucial, Take Home point:</u>

You are at a minimum if every available step is uphill