Merit functions:

- We have a constrained problem and wish to do line search - When does a step produce an improvement?

  - Tricky, because an improved objective might come at the cost of a weaker constraint.

Strategy:

- A merit function ~~scores to~~ balances objective, constraint scores

Merit functions have the form

$$\phi(x;\mu) = f(x) + \mu\left[\text{some term in constraints}\right] + \left[\text{occasionally, a term in equality constraints}\right]$$

An exact merit function has the property that

- ~~if~~ there is a μ^* st for any $\mu > \mu^*$, any local soln of the problem is a local minimizer of $\Phi(x;\mu)$.

For the problem

$$\min f$$
$$\text{st} \quad c(x) = 0$$
$$d(x) \leq 0$$

The merit function

$$\phi(x;\mu) = f(x) + \mu \sum_i |c_i(x)| + \mu \sum_i \cancel{\max}\ \cancel{\min}\ \max(0, d_i)$$

is <u>exact</u>.

The Maratos effect:

Consider

$$\min \; 2(x^2+y^2-1) \quad - x$$

$$\text{st} \quad x^2+y^2-1=0$$

We can check that $(1,0)$ is a soln.

our iterates are of the form

$$x^k = (\cos\theta, \sin\theta)$$

steps must be tangent to constraint; we might get

$$p^k = \sin\theta \begin{pmatrix} \sin\theta \\ -\cos\theta \end{pmatrix}$$

How fast do these steps approach the right point?

$$\|x_k + p_k - x^*\|_2 = 2\sin^2\theta/2$$

$$\|x_k - x^*\|_2 = 2|\sin\theta/2|$$

this means

$$\frac{\|x_k + p_k - x^*\|_2}{\|x_k - x^*\|_2^2} = \frac{1}{2}$$

!

which means the steps approach x^* rather well.

BUT

$$f(x^k + p^k) = \sin^2\theta - \cos\theta > -\cos\theta = f(x^k)$$

AND $\quad c(x_k + p_k) = \sin^2\theta > c(x_k) = 0$

!

So, if we require the merit fn. to go down, we have a problem.

- Options
  - Notice this is related to non-linearity of constraints and deal only w/ linear constraints
  - " , But now correct step by moving back toward constraints
  - Allow merit fn to increase on occasion - non-monotone strategy.