D

$P(D \mid \theta) \leftarrow \mathcal{L}(\theta)$

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}} \ \mathcal{L}(\theta)$$

$$= \underset{\theta}{\operatorname{argmax}} \ \log \mathcal{L}(\theta)$$

$D = x_i$, Normal dist

θ = stand dev. μ - mean.

$$P(D|\theta) = p(x_1|\theta) . p(x_2|\theta) \ldots$$

$$\log P(D|\theta) = \log p(x_1|\theta) + \log p(x_2|\theta) + \ldots$$

$$p(x|\theta) = \frac{1}{\sqrt{2\pi}\,\theta} e^{-(x-\mu)^2/2\theta^2}$$

$$\log P(D|\theta) = -\frac{(x_1-\mu)^2}{2\theta^2} - \log\theta - \ldots$$

$$= -\left[\sum_i \frac{(x_i-\mu)^2}{2\theta^2}\right] - N\log\theta$$

$$\hat{\theta} = \sqrt{\frac{\sum_i (x_i-\mu)^2}{N}}$$

D - data

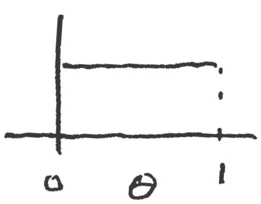
$p(\theta)$ - prior distribution

$$p(\theta|D) = \frac{p(D|\theta)\, p(\theta)}{p(D)} \quad !$$

(Bayes rule)

Coin   know NOTHING about $P(H) = \theta$

$P(\theta)$



flip   7 H,   3 T

$$\left(\frac{10!}{7!\,3!}\right) \theta^{7} (1-\theta)^{3} \qquad \theta^{H} (1-\theta)^{T}$$

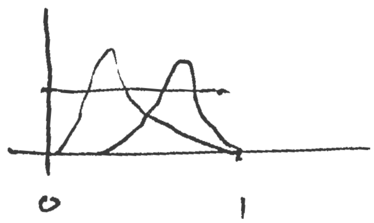
$$p(\theta|D) \propto P(D|\theta)\, p(\theta)$$

$$= \frac{P(D|\theta)\, P(\theta)}{\int P(D|\theta)\, P(\theta)\, d\theta}$$

θ – $p(H)$

N flips, h heads

$p(\theta)$ is a β distributions, $\alpha > 0, \beta > 0$



$$p(\theta | D) \propto P(D|\theta)\, P(\theta)$$

$$\frac{N!}{(N-h)!\,h!}\, \theta^{h} (1-\theta)^{N-h} \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} \cdot \theta^{(\alpha-1)} \cdot (1-\theta)^{(\beta-1)}$$

$$\propto (\ \ )\ \theta^{(\alpha+h-1)} (1-\theta)^{(N-h+\beta-1)}$$

$$= \frac{\Gamma(\alpha+\beta+N-1)}{\Gamma(\alpha+h)\,\Gamma(N-h+\beta)}\, \theta^{(\alpha+h-1)} (1-\theta)^{(N-h+\beta-1)}$$

$$\hat{\theta} = \frac{\alpha - 1 + h}{\alpha + \beta - 2 + N}$$ MAP estimate

$$p(\theta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \cdot \theta^{(\alpha-1)} e^{-\beta\theta}$$

$$p(D|\theta) = \prod_{i=1}^{N} \left(\frac{\theta^{n_i} e^{-\theta}}{n_i!} \right)$$

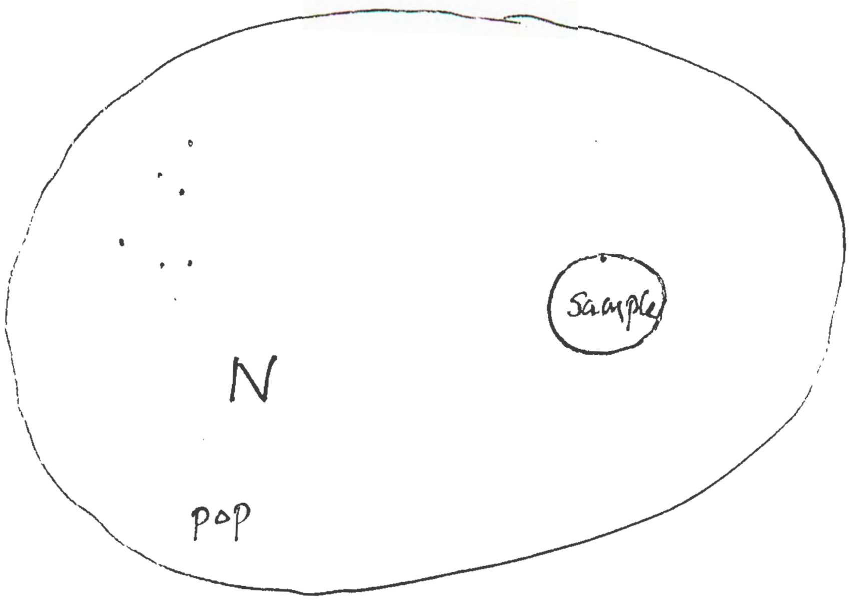
$$p(\theta|D) \propto \theta^{(\Sigma n_i + \alpha - 1)} \cdot e^{-(N+\beta)\theta}$$

$$= \frac{(N+\beta)^{(\Sigma n_i + \alpha)}}{\Gamma(\Sigma n_i + \alpha)} \cdot \theta^{(\Sigma n_i + \alpha - 1)} e^{-(N+\beta)\theta}$$

Normal prior x Normal likelihood

= Normal posterior

$$\mu_{posterior} = \frac{c \cdot (\text{measurement}) \cdot \sigma_{\pi}^2 + \mu_{\pi} \cdot \sigma_{likelihood}^2}{\sigma_{likelihood}^2 + c^2 \sigma_{\pi}^2}$$



$$\sum \text{values } P(\text{value})$$

$$\frac{1}{N}\sum x_i$$

popmean (X)

$$\text{sample} = \{ x_i \}$$

$$X^{(k)} = \frac{\sum_i x_i}{k}$$

$$E[X^{(k)}] = \frac{1}{k}\left[E[X_1] + E[X_2] + \cdots \right]$$

$$= E[X_1] = \text{popmean}$$

$$Var[X^{(k)}] = \frac{\text{popsd}^2}{k} \approx \frac{(\text{samplesd})^2}{k}$$