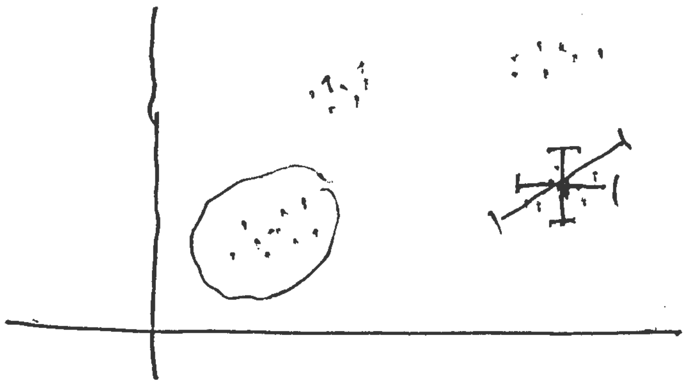
$\underset{\sim}{x}_i$



$$\mathrm{cov}(\{x_i\}, \{y\}) = \frac{\sum (x_i - \mathrm{mean}(x))(y_i - \mathrm{mean}(y))}{N}.$$

$$\mathrm{covmat}(\{\underline{x}\}) = \sum_u \frac{(\underline{x}_u - \mathrm{mean}(\underline{x}))(\underline{x}_u - \mathrm{mean}(\underline{x}))^T}{N}$$

d = dimension

i,j entry $= \mathrm{cov}(\underline{x}_i, \underline{x}_j)$

ith component

jth component

Positive semi-definite

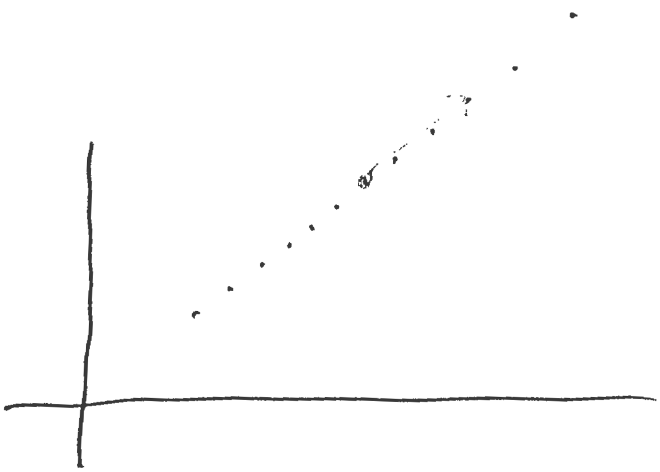
$$u^T \text{ covmat}(\{x\}) \, u \geq 0$$

in most cases > 0 Positive definite

u^T covmat( ) u

$$= u^T \left[\frac{\sum_i (\underline{x}_i - \text{mean})(\underline{x}_i - \text{mean})^T}{N} \right] u$$

$$= \frac{\sum_i [u^T (x_i - \text{mean})(x_i - \text{mean})^T u]}{N}$$



$$u^T(\underset{\sim}{x}_i - \text{mean}) = 0.$$

normal

for <u>any</u> $\underset{\sim}{x}_i$

$\underline{x}_i$

$$\underline{u}_i = A\underline{x}_i + \underline{b},$$

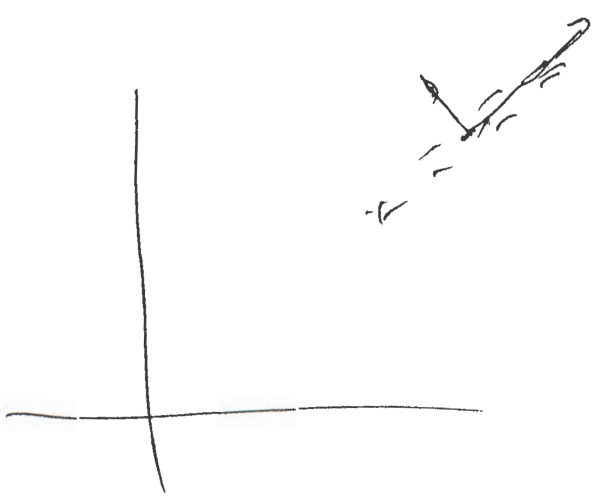
$$\text{mean}(\underline{u}) = A\,\text{mean}(\underline{x}) + \underline{b}$$

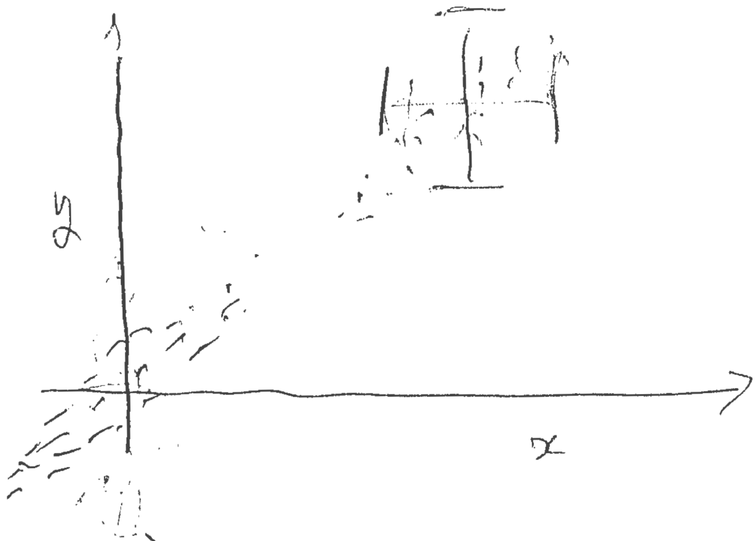
covmat(u)

$$= \text{covmat}(\{A x + b\})$$

$$= \frac{\sum_i (A x_i \cancel{+b} - A\,\text{mean}(x) \cancel{-b})(\qquad)^T}{N}$$

$$= \underline{A\,\text{covmat}(x) A^T}$$





$$\begin{pmatrix} \mathrm{cov}(x,x) & \cdot\ \cdot \\ \mathrm{cov}(x,y) & \mathrm{cov}(y,y) \end{pmatrix}$$

M

square
symmetric

$$\underbrace{\overset{d}{d\,|M}v = \lambda v}$$ ← eigenvector

d distinct eigenvectors

v_i

$$v_i \cdot v_j = \begin{cases} 1 & i = j \\ 0 & \text{otherwise} \end{cases}$$

$$U = \begin{bmatrix} v_1 & \cdots & v_d \end{bmatrix}$$

$$\boxed{\begin{array}{c}\text{orthonormal} \\ U^T U = U U^T = I_d\end{array}}$$

$$\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_d \end{bmatrix}$$

$$MU = U\Lambda$$

$$\begin{bmatrix} v_1 & \cdots & v_d \end{bmatrix} \qquad \begin{bmatrix} \lambda_1 v_1 & \lambda_2 v_2 & \cdots & \lambda_d v_d \end{bmatrix}$$

$$U^T M U = \Lambda$$

$$U^T \text{covmat}(x) U = \Lambda$$

$$\underline{n}_i = A \underline{x}_i + \underline{b}$$

$$\text{covmat}(n_i) = A \text{ covmat}(x) A^T$$

$$m_i = \underline{U}^T \underline{x}_i + b$$

$$[v_1 \ v_2 \ldots v_d] = U_a$$

$$[v_2 \ v_1 \ldots v_d] = U_b$$

$$\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_d \end{bmatrix}$$