$$U^T \operatorname{covmat}(x)\, U = \Lambda$$

$$\underline{n}_i = A\, \underline{x}_i + \underline{b}$$

$$\operatorname{covmat}(n_i) = A \operatorname{covmat}(x)\, A^T$$

$$m_i = \underline{U}^T \underline{x}_i + b$$

$$[v_1 \;\; v_2 \; .. \; v_d] = U_a$$

$$[v_2 \;\; v_1 \; .. \; v_d] = U_b$$

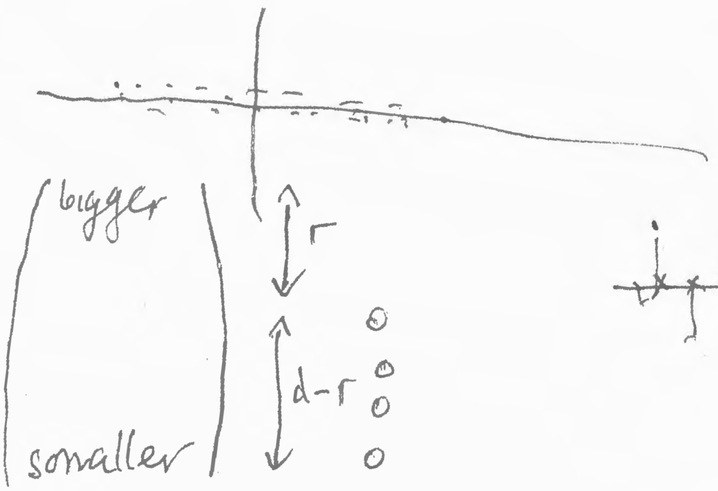
$$\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_d \end{bmatrix}$$

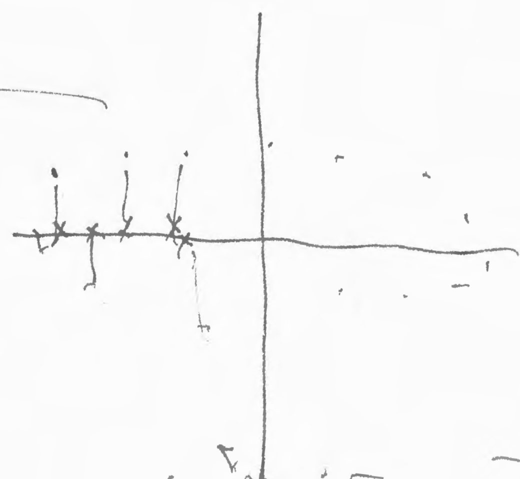
x_i

$\underline{u}_i = (\underline{x}_i - \text{mean}(x))$ ← translated

$U^T \text{covmat}(x) U = \Lambda$

$\underline{n}_i = U^T \underline{u}_i = U^T(\underline{x}_i - \text{mean}(x))$





$$\Pi^r = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 0 \end{pmatrix}$$

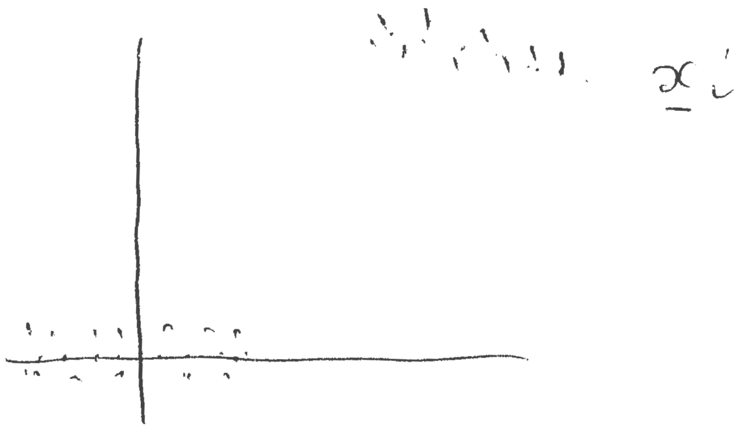
$$\underline{x}_i^{\text{smoothed}} = U\left[\Pi^r U^T(\underline{x}_i - \text{mean}(x))\right] + \text{mean}(x)$$

$$x_i^{\text{smoothed}} = U\left[\text{vector}\right] + \text{mean}(x)$$

$$= \begin{bmatrix} \underline{u}_1^e & \underline{u}_2^e & \cdots & \underline{u}_r^e & \underline{u}_{r+1}^e & \cdots & \underline{u}_d^e \end{bmatrix} \begin{array}{c} \begin{bmatrix} \vdots \\ 0 \end{bmatrix} \end{array} \begin{array}{l} \updownarrow r \\ \updownarrow d-r \end{array} + \text{mean}(x)$$

principal components

$$\sum_i \| \underline{x}_i - \underline{x}_i^{\text{smoothed}} \|^2$$



$$\sum_{\text{data}}^{N} \| x_i - x_i^{\text{smooth}} \|^2 = \sum_{j=r+1}^{d} \lambda_j$$