

Markov $P(\{|X| \geq a\}) \leq \frac{E[|X|]}{a}$

$$P(\{|X - E[X]| \geq a\}) \leq \frac{\text{Var}[X]}{a^2}$$

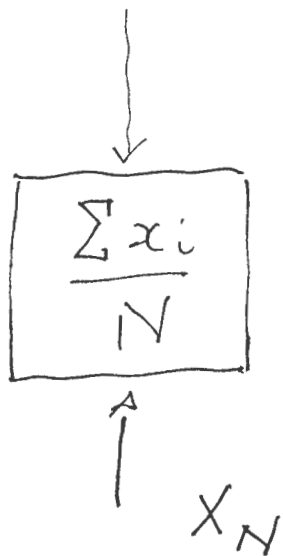
$$U = (X - E[X])^2$$

$$P(\{|U| \geq w\}) \leq \frac{E[|U|]}{w}$$

$$w = a^2$$

$$P(\{|X - E[X]| \geq a\}) = P(\{|U| \geq a^2\}) \\ \leq \frac{E[(X - E[X])^2]}{a^2}$$

IID x_i of X



$$\begin{aligned} E[X_N] &= \frac{1}{N} E \left[\begin{array}{l} \text{first sample} \\ + \text{second sample} \end{array} \right] \\ &= \frac{1}{N} \left(E[\text{first}] + E[\text{second}] + \dots \right) \\ &= E[X] \end{aligned}$$

—————→

$$\varepsilon > 0$$

$$\lim_{N \rightarrow \infty} P(\{ |X_N - E[X]| \geq \varepsilon \}) = 0$$

$$\underline{\text{Var}(X) = \sigma^2}$$



$$\begin{aligned} \text{var}(X_N) &= \text{Var}\left(\frac{\sum x_i}{N}\right) \\ &= \frac{1}{N^2} \cdot \text{Var}(\sum x_i) \\ &= \frac{1}{N^2} \cdot N\sigma^2 \end{aligned}$$

$$E[X_N] = E[X]$$

$$\text{Var}[X_N] = \frac{1}{N} \sigma^2$$

$$P(\{ |X_N - E[X]| \geq \varepsilon \}) \leq \frac{\sigma^2}{N \varepsilon^2}$$

$$\lim_{N \rightarrow \infty} P(\{ |X_N - E[X]| \geq \varepsilon \}) = 0$$

$$\lim_{N \rightarrow \infty} P(\{ |X_N - E[X]| < \varepsilon \}) = 1$$

18 r , 18 b , 10

$$1P(r) - 1P(\bar{r}) = 1 \cdot \frac{18}{37} - \frac{19}{37}$$
$$= -\frac{1}{37}$$

$$-\frac{2}{38} \approx \frac{1}{19}$$

$$-\frac{3}{39} = -\frac{1}{13}$$