Markov $\quad P(\{|X| \geq a\}) \leq \dfrac{E[|X|]}{a}$

$$P(\{|X - E[X]| \geq a\}) \leq \frac{Var[X]}{a^2}$$

$$U = (X - E[X])^2$$

$$P(\{|U| \geq w\}) \leq \frac{E[|U|]}{w}$$

$w = a^2$

$$P(\{|X - E[X]| \geq a\}) = P(\{|U| \geq a^2\})$$

$$\leq \frac{E[(X - E[X])^2]}{a^2}$$

IID x_i of X

$$\boxed{\frac{\sum x_i}{N}}$$

X_N

$$E[X_N] = \frac{1}{N} E\left[\text{first sample} + \text{second sample} \ldots\right]$$

$$= \frac{1}{N}\left(E[\text{first}] + E[\text{second}] + \cdots\right)$$

$$= E[X]$$

$\varepsilon > 0$

$$\lim_{N \to \infty} P\left(\left\{ \left| X_N - E[X] \right| \geq \varepsilon \right\}\right) = 0$$

$$\underline{Var(X) = \sigma^2}$$

$$var(X_N) = Var\left(\frac{\sum x_i}{N}\right)$$

$$= \frac{1}{N^2} \cdot Var\left(\sum x_i\right)$$

$$= \frac{1}{N^2} \cdot N\sigma^2$$

$$E[X_N] = E[X]$$

$$\mathrm{var}[X_N] = \frac{1}{N}\sigma^2$$

$$P(\{|X_N - E[X]| \geq \varepsilon\}) \leq \frac{\sigma^2}{N} \cdot \frac{1}{\varepsilon^2}$$

$$\lim_{N \to \infty} P(\{|X_N - E[X]| \geq \varepsilon\}) = 0$$

$$\lim_{N \to \infty} P(\{|X_N - E[X]| < \varepsilon\}) = 1$$

18 r , 18 b , 1 0

$$1\,p(r) - 1\,p(\bar{r}) = 1 \cdot \frac{18}{37} - \frac{19}{37}$$

$$= -\frac{1}{37}$$

$$-\frac{2}{38} \approx \frac{1}{19}$$

$$-\frac{3}{39} = -\frac{1}{13}$$