

1) (from Rabinowitz) Five fair dice are rolled. Find the probability that:

a) all five numbers are different

#(of cases where event occurs)/#(of cases)

#(of cases where event occurs)= $6 \times 5 \times 4 \times 3 \times 2$
(6 choices for first die, 5 for second, etc)

#(of cases)= 6^5

so $20/6^3=20/216$

b) at least two dice show the same number

#(of cases where event occurs)/#(of cases)=

$1 - \text{#(of cases where event does not occur)}/\text{#(of cases)} =$
 $1 - P(\text{all five nos different}) = 196/216$

c) at least one die shows a 6

$1 - P(\text{no die shows a six})$ and $P(\text{no die shows a six}) = 5^5/6^5$

2) Each firing of a missile at a target has a .21 probability of striking the target. The outcome of each firing is independent of the outcome of any others.

a) What is the probability of destroying the target with three shots?

~~$P(1 \text{ shot hits}) + P(2 \text{ shots hit}) + P(3 \text{ shots hit}) = 0.79^2 \cdot 0.21 + 0.79 \cdot 0.21^2 + 0.21^3$~~



b) I keep firing until I destroy the target. What is the expected number of shots I have to fire?

this is geometric, with $p=0.21$ so $E=1/p=1/0.21$ which is very slightly less than 5

3) (from Durrett) Roll two dice. Let A ="the sum is even" and B ="the sum is divisible by 3" (i.e. $B=\{3, 6, 9, 12\}$). Are A and B independent? explain

$P(A \cap B) = 2/36$; $P(A) = 1/2$; $P(B) = 4/12$; so $P(A \cap B)$ is not $P(A)P(B)$ so not independent.

4) Roll two dice. Let A ="The first die is odd" and B ="the second die is odd" and C ="the sum is odd". Show these events are pairwise independent, but not independent

obvious that A, B are pairwise independent; obvious that B, C are pairwise independent. $P(A \cap B \cap C) = 0$, which is not $P(A)P(B)P(C)$

5) (from Durrett) You want to invent a game where the player bets \$1, and rolls two dice. If the sum is 7, the player wins \$ k , and otherwise loses their bet.

a) What value of \$ k makes the game fair?

did this in class

b) If $k=1$, what is the expected value of the game to the player?

did this in class