


1) (From Durrett) A random variable has $P(X=x)=x/15$ for $x=1, 2, 3, 4, 5$, and $P(X=x)$ is 0 for all other values. What is the mean and variance of X ?

mean= $1.1/15+2.2/15+3.3/15+4.4/15+5.5/15=52/15$ which is about 3.5 

variance= $E(X^2)-E(X)^2$

and you can plug in the numbers

2) In a group of five items, two are defective. Find the distribution of N , the number of draws we need to find the first defective item. Find the mean and variance of N

this depends on whether we replace or not (question is badly set).

Assume we replace: then $p(\text{defective})=2/5$ and the distribution is geometric so

$P(1 \text{ draw})=2/5$

$P(2 \text{ draws})=(3/5)(2/5)$

$P(3 \text{ draws})=(3/5)^2(2/5)$

etc. Then the mean is $1/p$ so 2.5 and the variance is $(1-p)/p^2=(3/5)/(4/25)$

3) Can we have a random variable with $E[X]=3$ and $E[X^2]=8$?

4) Suppose a random variable X takes the values 1, 2, 3 and no other values. Suppose $E[X]=2.5$. What are the smallest and largest possible values for the variance?

6) A couple decides to keep having children until they have one of each gender. Ignore the possibility of twins, and assume that each trial is independent, and the probability of each gender is the same. What is the expected number of children that they will have?

7) Suppose we roll a red die and a green die. What is the probability that the number on the green die is larger than the number on the red die?