CS 498 Probability and Statistics for Computer Science Undergraduates

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Midterm Exam

| NAME | NETID |
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Instructions: 1) Please fill in your name and netid
2) Then fill in the right answers and return.

Information: You have 50 mins. There is a total of 22 marks. Good luck.
By submitting this exam for grading, you are asserting that the answers are your own unaided work.

| Question | Out of | Marks |
| :---: | :---: | :---: |
| 1 | 7 |  |
| 2 | 3 |  |
| 3 | 3 |  |
| 4 | 4 |  |
| 5 | 5 |  |

Question 1: You perform the following experiment: you take a six-sided die, and roll it. If the number that comes up is six, you stop; otherwise you repeat.
a) What is the distribution of the number of times you roll the die?
b) What is the expected number of rolls? (3)
c) Recall Chebyshev's inequality says that, for a random variable $X$
$P(\{|X-m e a n(X)|>=k \operatorname{std}(X)\})<=1 / k^{\wedge} 2$

Now use Chebyshev's inequality to give an upper bound on the probability that you will roll the die 100 or more times (3)

Question 2: In a group of 320 high school graduates, only 160 went to college, but 100 of the 170 men went to college. How many women did not go to college?

Question 3: Three couples that are invited to dinner will independently show up with probabilities $0.9,0.8$ and 0.75 . Let N be the number of couples that show up to dinner. Calculate the probability distribution of $N$. (3)

Question 4: Five people play the game of "odd-man-out" to determine who pays for a meal. In this game, each person flips a coin. If one person's coin comes up different to all others (i.e. there is one H and four T's or there is one T and four H's), then that person pays. Otherwise, everyone flips again. They go on doing this until someone is chosen. What is the expected number of times they must flip before they know who should pay? (4)

Question 5: A six-sided die has one red face, and all other faces are painted green. You roll this die 100 times. Let the random variable $X$ be the count of the number of times you see a red face.
a) What is the mean of $X$ ? (1)
b) What is the standard deviation of $X$ ? (1)
c) Recall Chebyshev's inequality says that, for a random variable $X$

$$
P(\{|X-m e a n(X)|>=k \operatorname{std}(X)\})<=1 / k^{\wedge} 2
$$

Now use Chebyshev's inequality to give an upper bound on the probability that you will see 100 red faces in 100 rolls (3)

