THE ZONAL METHOD FOR CALCULATING LIGHT INTENSITIES IN THE PRESENCE OF A PARTICIPATING MEDIUM

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ABSTRACT

The zonal method for calculating radiative transfer in the presence of a participating medium is applied to the generation of realistic synthetic images. The method generalizes the radiosity method and allows for emission, scattering, and absorption by a participating medium. The zonal method accounts for volume/surface interactions which have not been previously included, as well as volume/volume and surface/surface interactions. In addition, new algorithms, based on the hemi-cube formulation, are introduced for calculating the geometric factors required by the zonal method.

Categories and Subject Descriptors: I.3.3 [**Computer Graphics**]: Picture/Image Generation -- display algorithms, viewing algorithms; I.3.7 [**Computer Graphics**]: Three-Dimensional Graphics and Realism -- color, shading, shadowing, and texture

General Terms: Algorithms

Additional Key Words and Phrases: Clouds, light scattering, participating media, radiative transport, radiosity, zonal method.

1. INTRODUCTION

The basic radiosity method applies when the lighting in an environment is controlled primarily by the ideal diffuse reflection and emission of light from opaque surfaces [2,4]. The method has been extended to allow for directional light sources [10], and directionally reflecting and transmitting surfaces [6,11]. This paper extends the radiosity method to include the emission, absorption, or scattering of light by radiatively-participating media. Such media might include flames as light emitters, soot clouds as absorbers, or dust, smoke or fog as light scatterers. A scene with a light scattering/absorbing medium is shown in Fig. 1.

Figure 1 - Light scattering/absorption by a participating medium using the zonal method.

In a participating medium, light is emitted, absorbed or scattered on a volumetric basis. Thus, light traveling from one opaque surface to another can be attenuated, augmented, or redirected by the participating medium. The result is a multiplicity of transfer paths for light as in Fig. 1.

Blinn was the first to apply radiative transfer theory to the generation of images of participating media [1]. Blinn presented results for one dimensional models of media with low scattering albedo (i.e., with low effective reflectance). Recently, Max [8] has introduced an approximation of Blinn's method to efficiently generate images of clouds of complex geometry illuminated by sunlight. Max [9] has also applied Crow's shadow volume algorithm to the shadowing of participating media by opaque solids. Impressive images of atmospheric haze can be generated. Volume/volume interactions and surface/volume reflections are not included.

Kajiya and Von Herzen [7] built on the ideas introduced by Blinn, and developed a more complete method for three dimensional models of media with unrestricted scattering properties. In a preprocessing step, the intensity in the

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participating medium is calculated by using the scattering equation of transfer. Direct input from a light source and from inter-volume scatter are included. In a second step, the intensity due to surface reflections is combined with the intensity of the participating medium by ray tracing. The method does not account for surface/volume interactions, nor for diffuse surface/surface reflections.

In this paper a general method is introduced which allows for all volume/volume, volume/surface, and surface/surface interactions. The method assumes isotropic, volumetric emission, absorption, and scattering by the participating medium, and ideal diffuse reflection from opaque surfaces. Directional (non-diffuse) lighting is allowed. The method is based on, and extends, the radiosity method. The method was developed for radiant heat transfer analysis by Hottel and Sarofim [5] and is known as the zonal method. The participating medium is discretized into small volumes (or zones), each with a uniform volume radiosity. Complex geometries with arbitrary reflecting surfaces are readily included.

In the following sections, the zonal method is described, extended, and applied to the calculation of light intensities in a scene.

2. BASIC CONCEPTS

In the zonal method, all surfaces are assumed to be opaque, ideal-diffuse emitters and/or reflectors of light. Scattering and emission within the participating medium is assumed to be isotropic (i.e. independent of direction). In this paper, it is further assumed that light absorbed in one wavelength band is not re-emitted in another. This leads to independent sets of equations for the light intensities within discrete wavelength bands.

2.1 Absorption - The intensity of light is defined as the radiant energy crossing an area per unit time, per unit area (projected on a plane perpendicular to the direction of travel), and per unit solid angle. When light travels through an absorbing medium, however, the intensity of light decreases along the path.

Consider a pencil of light of intensity I incident on a small volume of matter, dV, as in Fig. 2a. The energy absorbed by the volume per unit time is:

$$d^3P_{abs} = \kappa_a I \, d\omega \, dA \cos\Theta \, dx = \kappa_a I \, d\omega \, d^2V \quad (1)$$

where κ_a is the absorption coefficient (the fraction by which the intensity of the light is reduced by absorption as it passes through a unit distance in the volume), $d\omega$ is the solid angle of the pencil of light, $dA \cos\Theta$ is the illuminated surface area, Θ is the angle between the pencil of light and the normal to dA, and dx is the distance traveled through the volume. On the right side of Eq. (1), d^2V denotes the differential volume through which the light passes.

Next, consider a larger beam of light of uniform intensity I which has the same directional orientation as the original pencil but fully illuminates the volume dV, as shown in Fig. 2b. The energy absorbed per unit time is found by integrating Eq. (1) :

$$d^2P_{abs} = \kappa_a I \, d\omega \, dV \quad (2)$$

Finally, consider light of constant intensity I arriving uniformly from all directions, as shown in Fig. 2c. The energy absorbed per unit time is found by integrating Eq. (2) over 4π steradians to obtain:

$$dP_{abs} = 4\pi\kappa_a I dV \quad (3)$$

In this integration, all surfaces are presumed to be uniformly illuminated by light of intensity I. Equation (3) can also be written in terms of a directionally-uniform incident flux density H, which is equal to πI (or to πI_{ave} if the intensity is not directionally uniform) as

$$dP_{abs} = 4 \kappa_a H \, dV \quad (4)$$

The units of H are energy per unit time per unit area.

2.2 Emission - Emitted energy leaving a surface is expressed by the energy flux density, E, in energy per unit time per unit area. The energy per unit time, P_{em}, emitted by a surface of area A is EA (watts). The analogous derivation of P_{em} for a volume with energy flux density E requires care, and the interested reader is referred to the first chapter of [5] for details. Basically, from the laws of thermodynamics it can be shown that the energy emitted per unit time, dP_{em}, from a volume

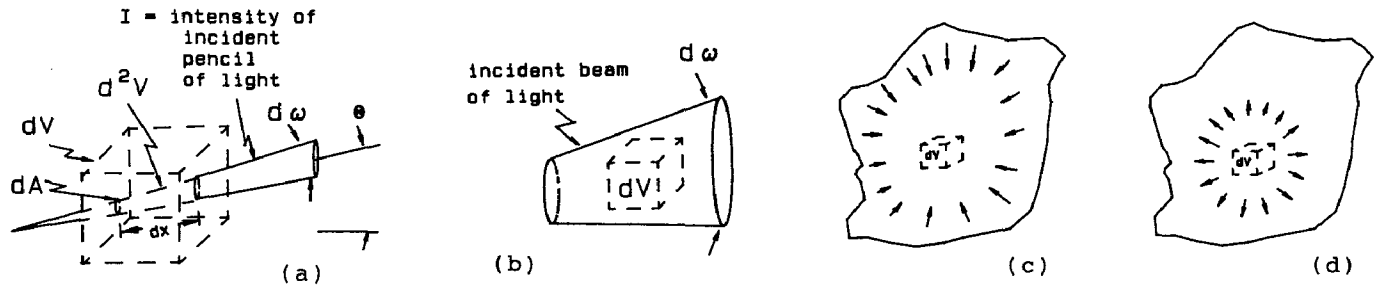

Figure 2 - Geometry of light absorption: (a) A small pencil of light intersecting a volume; (b) A beam of light encompassing a volume; (c) Uniform irradiation on a volume. (d) Geometry of light emission/scattering by a volume.

dV with emissive flux density E is:

$$dP_{em} = 4 \kappa_a E \, dV \quad (5)$$

where κ_a is the absorption coefficient. Since κ_a is the fraction by which intensity is reduced by absorption per unit length, the product $\kappa_a E$ has units of energy per unit time per unit volume. For isotropic emission, κ_a and E are independent of direction. Thus, the emitted light goes out uniformly in all directions, as sketched in Fig. 2d.

2.3 Scattering - Scattering redirects energy within a volume. Similar to the definition for absorption, let κ_s be the fraction by which the intensity of a pencil of light in a particular direction is reduced by scattering as it travels a unit distance. The total energy scattered per unit time, dP_{sc}, is:

$$dP_{sc} = 4 \kappa_s H \, dV \quad (6)$$

For isotropic scattering, the intensity of scattered light is uniform in all directions as sketched in Fig. 2d.

2.4 Volume Transmittance - The transmittance, τ, of a volume is a directional quantity which describes how much of the light on a given path will pass through the volume without being absorbed or scattered. For a pencil of light traveling a differential distance in a particular direction, the intensity is reduced by absorption and scattering by:

$$dI = -\kappa_t I \, dx, \quad \text{or,} \quad dI/dx = -\kappa_t I \quad (7a)$$

$$\text{where} \quad \kappa_t \equiv \kappa_a + \kappa_s \quad (7b)$$

Integration of Eq. (7) yields the intensity I(x) at a distance x into the medium:

$$I(x) = I_o \exp[- \int_0^x \kappa_t \, dx^*] \quad (8a)$$

where I_o is the initial intensity, and * denotes a dummy variable. Thus, the transmittance is given by:

$$\tau(x) = \exp[- \int_0^x \kappa_t \, dx^*] \quad (8b)$$

2.5 Volume Radiosity - The flux density leaving a volume element due to emission and scattering is defined as the volume radiosity B, given by:

$$4 \kappa_t B = 4 \kappa_a E + 4 \kappa_s H \quad (9)$$

The radiosity of a volume includes only the energy which has been emitted or scattered by the volume. Energy that is transmitted directly through the volume is not included.

By introducing the isotropic scattering albedo, defined as $\Omega \equiv \kappa_s/\kappa_t$, Eq. (9) can be written in the alternate form:

$$B = (1 - \Omega)E + \Omega H \quad (10)$$

Equation (10) is similar to the defining equation for the radiosity of a surface. The isotropic scattering albedo Ω appears in place of the diffuse reflectance ρ of the surface.

2.6 Variation of Intensity along a Light Path - The intensity of light reaching the eye depends on the attenuation of intensity along the light path due to absorption and scattering, and on the enhancement due to emission and in-scattering. In-scattering refers to the scattering of incident light into the direction of interest. Equation (7) describes attenuation along a path. Similarly, the enhancement can be described by:

$$dI/dx = \kappa_t J(x) \quad (11)$$

where J(x) is the scattered and emitted intensity originating from an infinitesimal volume of thickness dx at x. For isotropic emission and scattering J(x) is related to the volume radiosity by $J(x) = B(x)/\pi$. Defining $t = \int_0^x \kappa_t(x^*)dx^*$, and summing Eqs. (7) and (11), the following relation between the local intensity and the scattered/emitted intensity results:

$$dI/dt = -I(t) + J(t) \quad (12)$$

Integration of Eq. (12) yields:

$$I(t) = \exp(-t)\{ I_o + \int_0^t J(t^*)\exp(t^*) \, dt^*\} \quad (13)$$

3. GEOMETRIC FACTORS

In the radiosity method [4], linear algebraic equations for the surface radiosities are constructed by using form factors which depend on geometry alone. In the zonal method the concept of form factors is extended to include volumes.

3.1 Surface to Surface (SiSj) Factors - The geometry of two surfaces interchanging energy is shown in Fig. 3a. In the absence of a participating medium, the fraction of energy leaving a differential element dA_i and reaching a differential element dA_j is:

$$F_{dAidAj} = \cos\Theta_i \cos\Theta_j dA_j/(\pi s_{ij}^2) \quad (14)$$

An absorbing/scattering medium will reduce the amount of energy traveling directly from dA_i to dA_j, and a transmittance τ (as given in Eq. (8b)) must be included:

$$F_{dAidAj} = \tau(s_{ij})\cos\Theta_i \cos\Theta_j dA_j/(\pi s_{ij}^2) \quad (15)$$

The fraction of energy leaving a finite surface A_i of uniform radiosity B_i that arrives at a second finite surface A_j is found by integrating over the two surfaces:

$$F_{ij} = (1/A_i) \int_{Ai} \int_{Aj} \tau(s_{ij})\cos\Theta_i \cos\Theta_j dA_j dA_i/(\pi s_{ij}^2) \quad (16)$$

This differs from a conventional form factor by the inclusion of the transmittance τ inside the

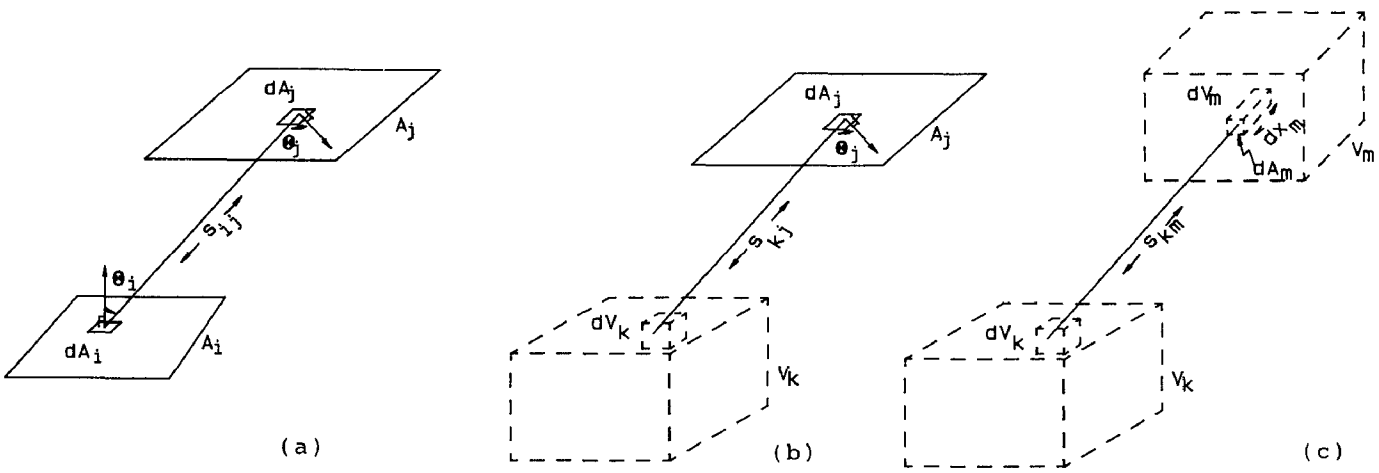

Figure 3 - Geometry of light interchange, (a) Between two surfaces; (b) Between a volume and a surface; and (c) Between two volumes.

integral. In the zonal method it is more convenient to use a factor SiSj given by $A_i F_{ij}$, or:

$$SiSj = \int_{Ai}\int_{Aj} \tau(s_{ij})\cos\Theta_i\cos\Theta_j dA_j dA_i/(\pi s_{ij}^2) \quad (17)$$

SiSj can be interpreted as the total energy reaching surface j from surface i, divided by the radiosity of surface i. Note that SiSj is equal to SjSi; this is consistent with the reciprocity requirements.

3.2 Volume to Surface (VkSj) Factors - The geometry of a volume exchanging energy with a surface is shown in Fig. 3b. The total energy scattered from and emitted by a volume is found by summing Eq. (5) and (6) and applying the definition given in Eq. (9). The fraction of energy emitted from or scattered by volume dV_k and reaching a surface dA_j, in the absence of any intervening medium, is the solid angle subtended by dA_j divided by 4π. The total energy d^2P_{kj} emitted or scattered by volume dV_k reaching dA_j is:

$$d^2P_{kj} = 4\kappa_t B_k dV_k \cos\Theta_j dA_j/(4\pi s_{kj}^2) \quad (18)$$

An intervening medium reduces the amount of energy reaching dA_j, and introduces the transmittance τ:

$$d^2P_{kj} = \tau(s_{jk})\kappa_t B_k dV_k \cos\Theta_j dA_j/(\pi s_{kj}^2) \quad (19)$$

The total energy emitted from or scattered by a finite volume V_k and arriving at a finite surface A_j is found by integrating Eq. (19) over the entire volume and the entire receiving surface. If the radiosity of the volume is taken as spatially uniform, a purely geometric factor arises. Analogous to the factor SiSj, the factor VkSj is defined as the total energy emitted from or scattered by V_k that arrives at A_j, divided by the radiosity of volume V_k, or:

$$VkSj = \int_{Vk}\int_{Aj} \tau(s_{jk})\kappa_t dV_k \cos\Theta_j dA_j/(\pi s_{kj}^2) \quad (20)$$

A factor SjVk can be defined which is the total energy leaving surface j (of uniform radiosity B_j), which is either scattered or absorbed by volume k, divided by the radiosity of surface j. The reciprocity relationship SjVk = VkSj holds.

3.3 Volume Volume (VkVm) Factors - The geometry of two volumes exchanging energy is shown in Fig. 3c. The energy d^2P_{km} emitted from or scattered by volume dV_k, and incident on volume dV_m, after attenuation by an intervening medium, is:

$$d^2P_{km} = \tau(s_{km})\kappa_{t,k} B_k dV_k \cos\Theta_m dA_m/(\pi s_{km}^2) \quad (21)$$

The fraction of this energy absorbed or scattered by dV_m is $\kappa_{t,m} dx_m$. The energy scattered or emitted by dV_k, which is absorbed or scattered by dV_m, is given by:

$$d^2P_{km} = \tau(s_{km})\kappa_{t,k} B_k dV_k \cos\Theta_m dA_m \kappa_{t,m} dx_m/(\pi s_{km}^2)$$
$$= \tau(s_{km})\kappa_{t,m}\kappa_{t,k} B_k dV_k dV_m/(\pi s_{km}^2) \quad (22)$$

The product $\cos\Theta_m dA_m dx_m$ for the parallelepiped shown in Fig. 3c is equal to dV_m. Any arbitrary volume can be approximated by a set of such parallelepipeds. Thus, the amount of scattered or absorbed energy depends only on the volume of dV_m, and not on its orientation.

The total energy scattered by or emitted from a finite volume V_k of uniform volume radiosity B_k, which is absorbed or scattered by a finite volume V_m, is found by integrating over the two volumes. Similar to the factors SiSj and VkSj, a factor VkVj appears which represents the total scattered and emitted energy leaving V_k which is absorbed or scattered by volume V_j, divided by the radiosity B_k, or:

$$VkVm = \int_{Vk}\int_{Vm} \tau(s_{km})\kappa_{t,m}\kappa_{t,k} dV_k dV_m/(\pi s_{km}^2) \quad (23)$$

4. RADIOSITY EQUATIONS

A set of simultaneous linear algebraic equations for the zonal method is derived in this section. This set couples the radiosities of surfaces and volumes, and can be formed by expressing the energy scattered or absorbed by each surface and volume in terms of the radiosities of the other surfaces and volumes in the environment.

The total energy leaving a surface i is given by the product B_iA_i, where B_i is the radiosity of the surface. B_i consists of emitted and reflected energy. The energy emitted by the area is E_iA_i. Energy incident on the surface comes partially from other surfaces and partially from other volumes in the environment. The energy incident on surface i from surface j is $B_j\underline{SjSi}$. The energy incident on surface i from volume k is $B_k\underline{VkSi}$. Therefore, a complete equation for the radiosity B_i of a surface is:

$$B_iA_i = E_iA_i + \rho_i\{\Sigma\, B_j\underline{SjSi} + \Sigma\, B_k\underline{VkSi}\} \qquad (24)$$

where the first term represents the emitted energy, the remaining terms represent reflected energy, and ρ_i is the surface reflectance.

The complete equation for the radiosity B_k of a volume is:

$$4\kappa_tB_kV_k = 4\kappa_aE_kV_k + \varrho_k\{\Sigma\, B_j\underline{SjVk} + \Sigma\, B_m\underline{VmVk}\} \qquad (25)$$

On the left is the total energy emitted or scattered from a volume. The term $4\kappa_aE_kV_k$ is the emitted energy. The term $\varrho_kB_j\underline{SjVk}$ is the energy leaving a surface A_j which is scattered by the volume V_k. The energy emitted by or scattered from another volume m which is scattered by volume k is $\varrho_kB_m\underline{VmVk}$.

For a system of s surfaces and v volumes, equations of the form (24) for surfaces and (25) for volumes result in s+v simultaneous equations for the s+v radiosities in the environment. Both volume and surface radiosities appear. In general the equations are strongly coupled. The s+v equations represent an energy-conserving model of a lighting environment, and can be applied monochromatically or to discrete wavelength bands.

5. COMPUTATIONAL CONSIDERATIONS

5.1 Input Data - The geometry and properties of surfaces can be specified in the zonal method in the same way they are for the basic radiosity method. New data structures are required to store the attributes of volumes of participating media. A simple method is to define each volume containing a participating medium as a rectangular prism. Octree data structures used for accelerated ray tracing [3, 12] could also be used. For each volume of participating medium, E, κ_t, and ϱ must be specified.

5.2 Finding Factors - The zonal method can be implemented by calculating the factors $\underline{SiSj}$, $\underline{VkSj}$ and $\underline{VkVm}$ using variations of the hemi-cube algorithm [2]. The algorithm calculates form factors by using a depth buffer to project surfaces onto a half cube. In the zonal method, reciprocity relations are used to reduce calculations.

Surface to Surface Form Factors - The factor $\underline{SiSj}$ is given by Eq. (17). This is approximated by:

$$\underline{SiSj} \approx A_i \int_{Aj} \tau(s_{ij})\cos\Theta_i\cos\Theta_j dA_j/(\pi s_{ij}^2) \qquad (26)$$

where it is assumed that the surfaces are sufficiently finely divided so that the integral is nearly constant over the surface A_i. Equation (26) is further approximated by :

$$\underline{SiSj} \approx A_i\Sigma_p\, \tau(s_{ij}(p))\cos\Theta_i\cos\Theta_p dA_p/(\pi s_{ip}^2) \qquad (27)$$

where the sum is over all of the hemi-cube grid cells p through which area A_j is visible, as shown in Fig. 4, and $s_{ij}(p)$ is the distance from surface A_i to the point on A_j which is visible through grid cell p. The terms in this summation, with the exception of the factor τ, are the same as for the basic hemi-cube algorithm. The value of τ needs to be approximated for each grid cell. For a medium completely filling the region between A_i and A_j, as shown in Fig. 4a, with spatially uniform properties, τ is given by:

$$\tau(s_{ij}(p)) = \exp\{-\kappa_t\sqrt{(x^2 + y^2 + z^2)}\} \qquad (28)$$

where x, y, and z are the coordinates of a point on surface A_j as seen through grid cell p in a rectangular, undistorted, coordinate system based on surface A_i. A transformation is made to a perspective coordinate system in which:

$$x' = x/z \quad , \quad y' = y/z \quad , \quad z' = \alpha + \beta/z \qquad (29)$$

where α and β are arbitrary constants. Equation (28) can be rewritten:

$$\tau(s_{ij}(p)) = \exp\{-(\kappa_t(\beta/(z'-\alpha))\sqrt{(x'^2+y'^2+1)}\} \qquad (30)$$

Let $d_p = \sqrt{(x'^2 + y'^2 + 1)}$. Thus, Eq. (27) becomes:

$$\underline{SiSj} \approx A_i\,\Sigma_p \exp\{-(\kappa_t\beta/(z'-\alpha))d_p\} [\cos\Theta_i\cos\Theta_p dA_p/(\pi s_{ip}^2)] \qquad (31)$$

In Eq. (31) the quantity d_p and the quantity in square brackets are calculated once for each grid cell. As the factors for a particular surface A_j are calculated, the depth buffer is consulted to determine z' for A_j for each grid cell so that τ can be calculated. Pseudo-code for calculating factors after the depth buffer is filled is:

```
FOR each grid cell buffer location p BEGIN
    IF object[p] != null BEGIN
        z = β/(depth_buffer[p]-α);
        τ = exp(- κt * z * d[p]);
```

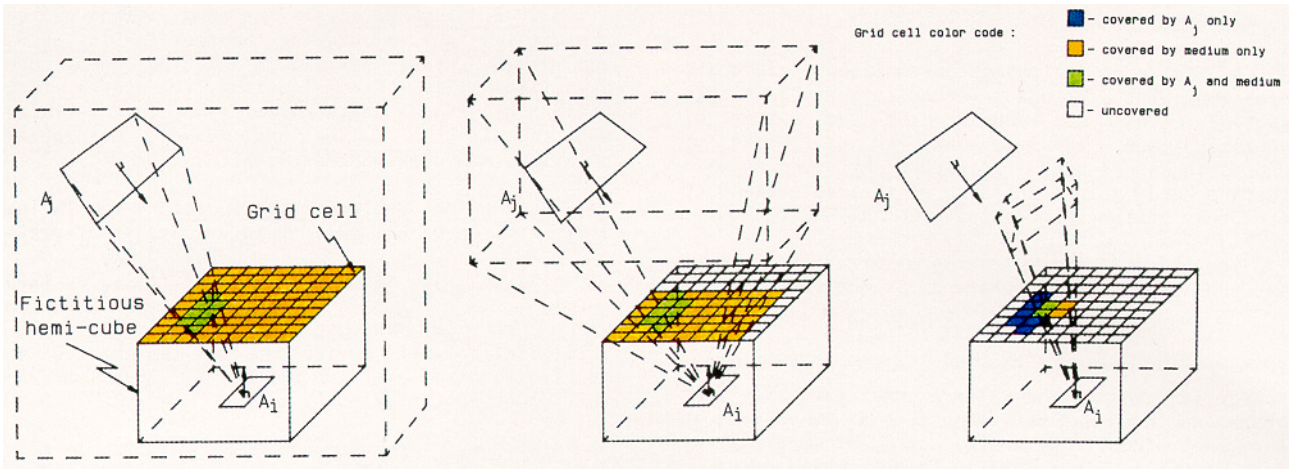

Figure 4 - Surface to surface form factor determination when the participating medium (a) surrounds both surfaces and the region in between; (b) surrounds one surface; and (c) lies between two surfaces.

```
                ss[i.object[p]) += τ * bracketed_factor[p];
            END
        END
        FOR j = 1 to number of surfaces
            ss[i.j] *= Ai ;
```

When the medium occupies only part of the space, as shown in Figs. 4b and 4c, or if it is modeled as several small regions each of which has a different value for κ_t, a modification of the foregoing procedure is required. A value of τ is calculated for each region r, and the total value of τ is the product of these values. To determine each τ_r, before the factors are calculated but after the depth buffer is filled, the depths of the surfaces bounding each region r of a different κ_t at each grid cell must be determined. Two buffers are maintained for each region - a front buffer (fb) and a back buffer (bb). Using the values in fb and bb, an attenuation factor for each grid cell is calculated as follows:

```
Initialize atten_buffer to 0.;

FOR each region of medium r BEGIN
  fill front buffer fb;
  fill back buffer bb;
  FOR each buffer location p BEGIN
    IF ( fb[p] < depth_buffer[p]) THEN
      IF (fb[p] > 0) zfront = β/(fb[p]-α);
      ELSE zfront = 0;
      IF (bb[p] < depth_buffer[p])
          zback = β/(bb[p]-α);
      ELSE zback =
          β/(depth_buffer[p]-α);
      atten_buffer[p] +=
        κt * d[p] * (zback - zfront);
    END IF
  END FOR each p
END FOR each r

FOR each buffer location p BEGIN
    IF (atten_buffer[p] == 0.)
        atten_buffer[p] = 1.;
    ELSE atten_buffer[p] =
        exp ( - atten_buffer[p]);
END FOR
```

The pseudo-code for calculating factors after filling the depth buffer and the attenuation buffer becomes:

```
FOR each grid cell buffer location p
  IF (object[p] != null)
    ss[i.object[p]) += atten_buffer[p]
        * bracketed_factor[p];
FOR j = 1 to number of surfaces
    ss[i.j] *= Ai ;
```

Volume to Surface Factors VkSj - The factor VkSj given by Eq. (20) is approximated by:

$$\text{VkSj} \simeq V_k \kappa_t \sum_p \tau(s_{kj}(p))[\cos\Theta_p dA_p/(\pi s_{kp}^2)] \quad (32)$$

As before, τ is evaluated either for a single medium or for several media, and the quantity in brackets is calculated once for each grid cell. The factors VkSj are calculated similarly to the factors SiSj, except that a full-cube rather than a hemi-cube is used (see Fig. 5a).

Volume to Volume Factors VkVm - Volume to volume factors VkVm are given by Eq. (23). If the volumes are sufficiently subdivided the factor VkVm can be approximated by:

$$\text{VkVm} \simeq \tau(s_{km})\kappa_{t,k}\kappa_{t,m}V_k V_m/(\pi s_{km}^2) \quad (33)$$

where the distance between two volumes is taken as the distance between their center points. The visibility of one volume to another is determined quickly by using the depth buffer previously filled in the determination of VkSj. For each volume V_m the grid cell on which the center of V_m will be projected is determined, as shown in Fig. 5b. Visibility is assessed by comparing the depth of

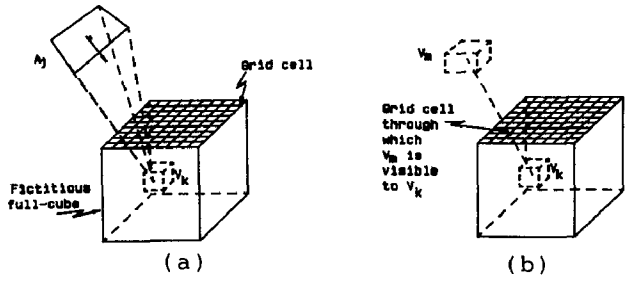

Figure 5 - Form factor determination (a) Volume to surface. (b) Volume to volume.

the volume center to the value stored in the depth buffer.

Once visibility is determined, the distance between the centers is calculated, both for use in the denominator in (33) and for calculating τ. (If many media are present the front buffer / back buffer method is repeatedly used to determine τ.)

5.3 Calculating Radiosities - After the geometric factors for an environment have been found, a system of equations for the radiosities of surfaces and volumes is formed using Eqs. (24) and (25). The equations are diagonally dominant, and the Gauss-Seidel iterative solution method can be used.

5.4 Rendering - The determination of the intensity of an individual pixel on the screen is diagrammed in Fig. 6. The intensity along the ray shown is determined by accounting for the attenuation, in-scattering and emission. It is assumed that there is no interference between light rays. The viewing intensity of each surface and volume along the path is determined separately and the results summed.

In an environment composed of opaque surfaces, there is only one visible surface along each ray. The visible surfaces are determined using the depth buffer algorithm. The intensity contribution of the visible surface is its intensity (equal to its radiosity divided by π) attenuated by the value of τ along the ray from the surface to the eye. The value of τ along the path is determined by the same front buffer/ back buffer method used to find τ in the determination of geometric factors.

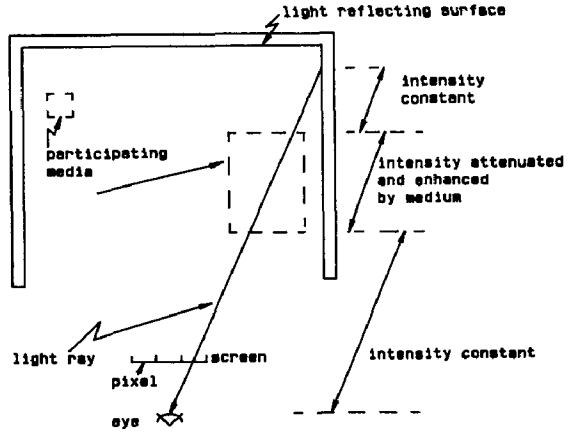

Figure 6 - Pixel intensity calculation.

For the intensity contributions of volumes, it is necessary only to find the intensity leaving the front face of the volume when the volume is isolated, and then attenuate that intensity by the appropriate value of τ. Consider a hypothetical light path through such an isolated volume V_k of radiosity B_k, for a particular screen pixel, as shown in Fig. 7. The points on the surface of the volume pierced by the rays passing through the centers of the pixels can be determined by finding the depths of the surfaces bounding the volume and storing the results in a front buffer and a back buffer.

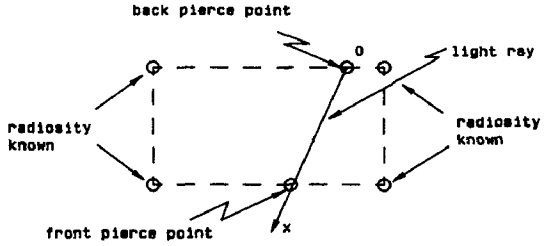

Figure 7 - Intensity calculation for an isolated volume.

Once the path through the volume has been found using the front buffer/back buffer, the intensity $I_k(x)$ is calculated using Eq. (13). For an isolated volume, I_0 is zero, $J(t)$ is the radiosity B_k divided by π, and t varies from 0 to $\kappa_t x$. Equation (13) yields:

$$I_k(x) = \exp(-\kappa_t x) \int_0^{\kappa x} (B_k(t)/\pi)\exp(t)\, dt \quad (34)$$

B_k is assumed to vary linearly along the path through the volume, and varies from some value B_{kb} at $t = 0$ to B_{kf} at $t = \kappa_t x$. The values at the front and back of the volume are found by bilinear interpolation between discrete points on the surface of V_k. The radiosities at the discrete points are found by averaging the radiosities of the surrounding volumes. The foregoing procedure reduces the visibility of spatial discretization of the participating medium.

6. ILLUSTRATIVE EXAMPLES

Several images will be presented to illustrate the zonal method. The images in Figs. 1, 8, 9, 10 and 11 were calculated using a Digital Equipment Corporation VAX 11/780, at a resolution of 400 x 400. Individual and composite images were displayed using 24 bit, 1280 x 1024 resolution frame buffers-- a Raster Technologies Model One/380 and a Hewlett Packard 98720A.

The images shown in Fig. 12 were calculated at a resolution of 1000 x 1000 using a VAX 8700 and were displayed using the HP 98720A frame buffer. The environment displayed in Fig. 12 was discretized into 1146 surfaces and 2744 volumes. Calculating geometric factors and solving for radiosities required approximately 3 cpu hours. Rendering each image required approximately 3 additional cpu hours.

6.1 Volume Interactions - Four images of a black-walled cubical enclosure are displayed in Fig. 8. The enclosure is filled with an absorbing/isotropically-scattering participating medium which is illuminated by a planar rectangular ideal diffuse white light source on the ceiling. A constant value of $\kappa_t = (1/L)$ was used, where L is the length of one side of the enclosure. The upper and lower rows of images were computed using scattering albedoes of $\Omega = 0.8$ and $\Omega = 0.3$ respectively.

The zonal method was used to calculate the images on the left, and a single scatter approximation was used to obtain the images on the right. For a high scattering albedo, top row, volume/volume interactions are absent in the image on the right. Either the zonal method or Kajiya and Von Herzen's method can be used to obtain the correct image of the high albedo medium in the upper left. For the lower row of images for a low scattering albedo, volume/volume interactions can be ignored without affecting the final image. The zonal method and single scatter models are in close agreement. Either Kajiya and von Herzen's method or Max's version of Blinn's single scatter model can be used for correct images.

6.2 Surface/Volume Interactions - The presence of opaque surfaces in a participating medium may either increase the illumination of the medium when light is reflected from a surface into the medium, or it may decrease the illumination of the medium when the opaque surface lies between the medium and light sources. In addition, a dense participating medium can cast shadows, and alter the illumination of opaque surfaces.

The effect of light reflected from an opaque surface is illustrated in Fig. 9, which shows the same enclosure displayed in Fig. 8, but with one of the walls given a non-zero reflectance. Values of $\kappa_t = (1/L)$ and $\Omega = 0.8$ were used for all four images. The upper and lower rows of images correspond to enclosures with a red and a cyan wall, respectively. In the left column of images, the colored wall is to the left. In the right column, the view is through the colored wall. Clearly the presence of the reflecting wall affects the illumination of the participating medium, and the wall color bleeds into the medium.

The shadowing effect of opaque objects in a participating medium is shown in Fig. 10. Two red blocks have been placed in the black walled enclosure. Values of $\kappa_t = (1/L)$ and $\Omega = 0.7$ were used. The blocks cast shadows on the medium.

The shadowing effect of a medium on an opaque object is shown in Fig. 11. A white planar rectangular light source illuminates a small dense cloud with high scattering albedo which casts a shadow (with penumbra) on an opaque green surface. The cloud was modeled by assigning values of of κ_t to each sub-volume. The κ_t values varied from (10/L) at the center of the cloud to zero at the outer edge, where L represents the length of one edge of the green surface. A uniform value of $\Omega = 0.8$ was used.

6.3 Combined Interactions - Figures 1 and 12 display images in which surface/surface, surface/volume, and volume/volume interactions are all present. Constant values of $\kappa_t = .7/L$ and $\Omega = 0.95$ were used to define the participating medium. The light source outside the windows simulate outdoor lighting, with a small intense distant light source and a distributed, weaker, hemispherical source. Figures 12a and 12b show the same environment (with the same illumination) from different viewing positions. Figure 12c shows the environment with the color and position of the source illumination changed. Figure 12d shows the environment with the light source embedded within the participating medium. Clearly the zonal method allows the subtle interactions between surfaces and volumes to be included, without restrictions on view point or the location of the light source.

7. CONCLUSIONS

The zonal method for radiant transfer in participating media [5] has been applied to the generation of synthetic computer images. The method allows all volume/volume, volume/surface, and surface/surface interactions to be included for the case of isotropic participating media and diffuse reflecting surfaces. The method requires extensions of the hemi-cube algorithm to calculate form factors.

The zonal method is the first method to correctly render images of participating media for all possible values of the scattering albedo. Surface/volume interactions such as color bleeding and shadowing are included. The method is more complicated than the basic radiosity method, but generalizes that method.

The zonal method is both view independent and view dependent. The view independence arises because for isotropic scattering radiosities can be calculated once. The view dependence arises because the total intensity leaving any point in the medium varies with direction. The total intensity must be calculated by path integration after the viewpoint is specified.

The zonal method is a potentially very powerful method which warrants further development. Possible extensions of the present study include adaptive subdivision and substructuring of volumes, directional volume and surface scattering, and the use of a hardware depth buffer.

8. ACKNOWLEDGMENTS

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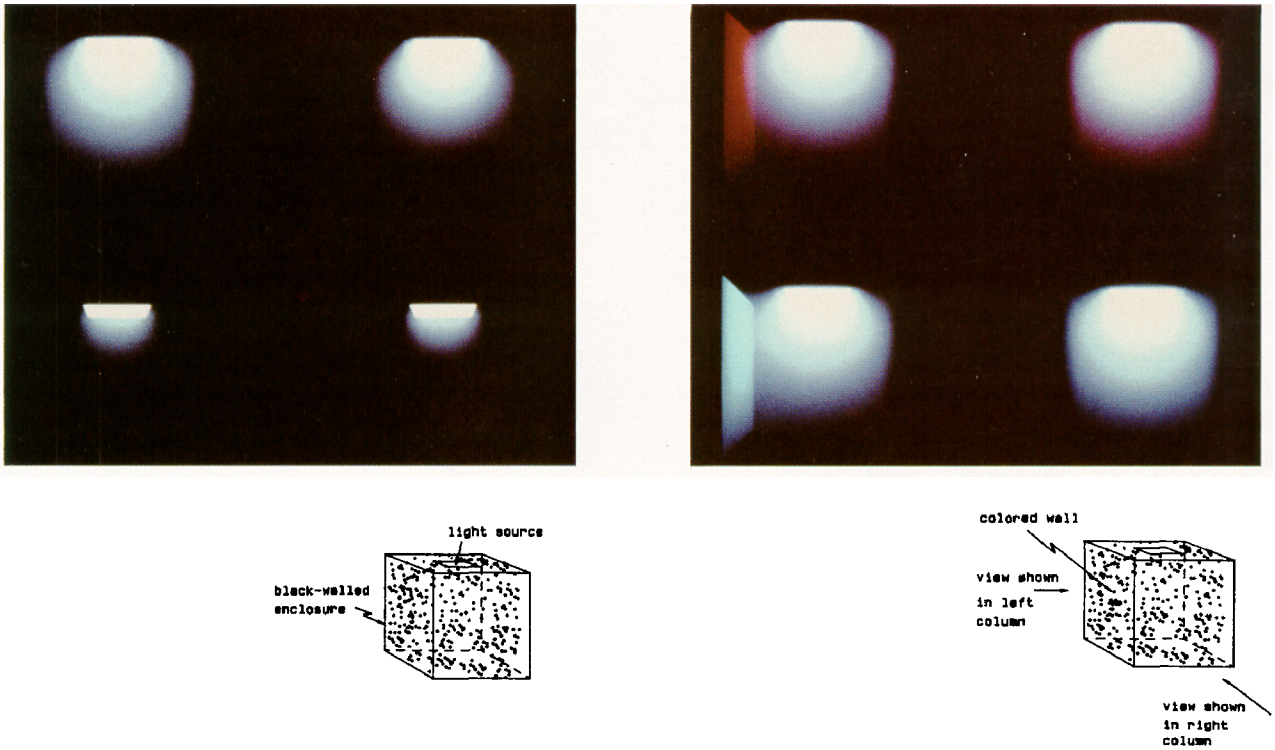
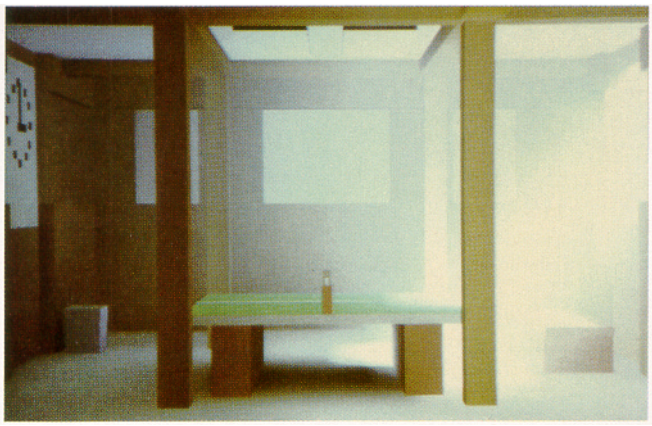
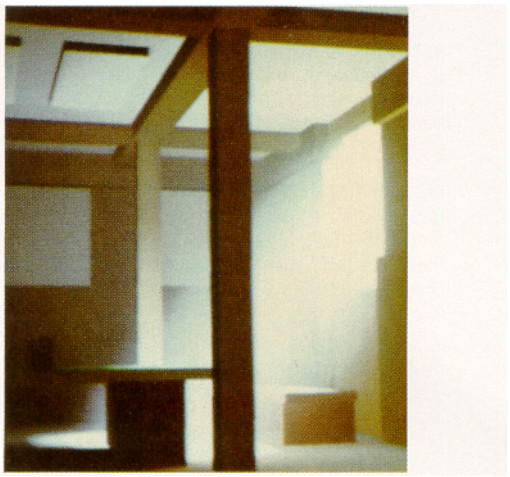


Figure 8 - Comparison of two solution techniques for a participating medium in a cubical enclosure. The top row corresponds to a high scattering albedo, the bottom row to a low scattering albedo. The left column shows the full solution, and the right column shows the single scatter solution.

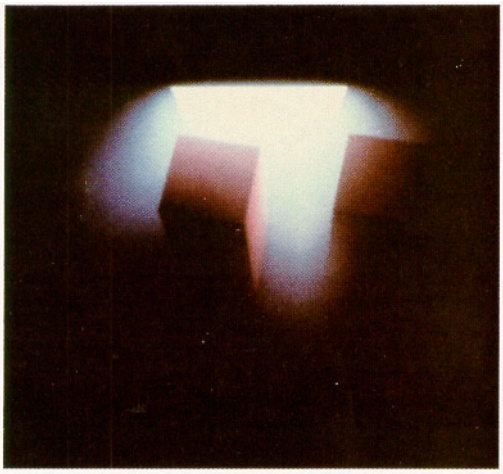
Figure 9 - Color bleeding from a wall to a participating medium in a cubical enclosure. The left column shows the colored wall to the left. The right column shows the view through the colored wall.

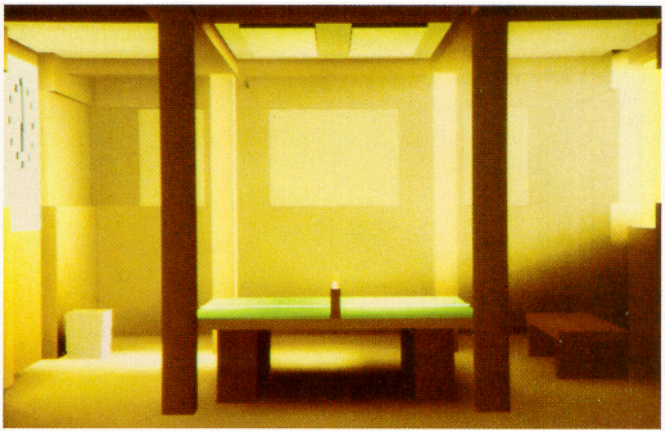


(12a)



(12b)





(12c)

Figure 10 - Shadowing of a participating medium by opaque solids.

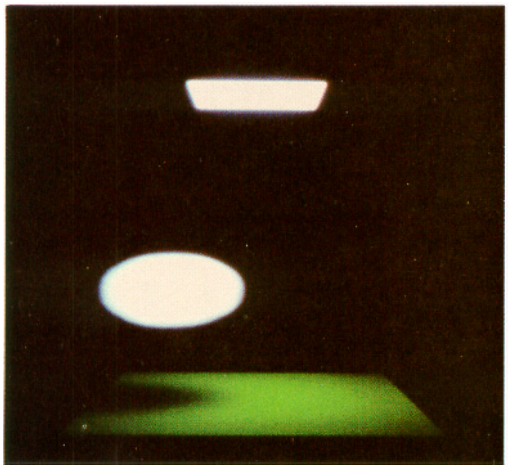
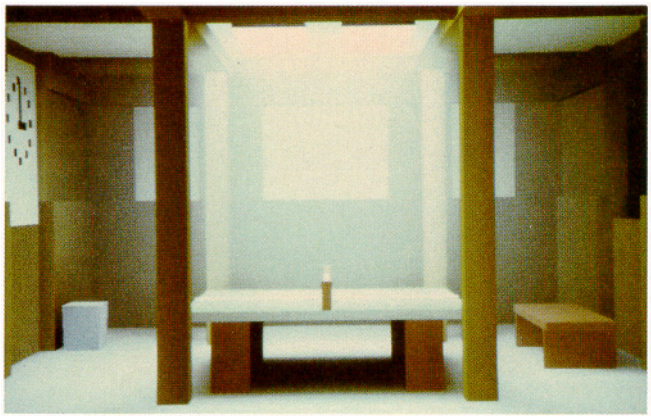


Figure 11 - Shadowing of an opaque solid by a participating medium.



(12d)

Figure 12 - Light scattering/absorption in a smoky room using the zonal method -- (a) afternoon, (b) afternoon, second view, (c) sunset, (d) late at night.